ALTERNATING CURRENTS

PART II

BY

C. G. LAMB, M.A.

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PREFACE

THE present is the second section of the notes on alternating currents intended as a guide to a student attending a three-term course on the subject. The aim is to give him a concise introduction to the main principles involved, so as to enable him to proceed to works on design such, for example, as Professor Miles Walker's *Specification and Design of Dynamo-Electric Machinery.*

The author must express his obligation to Professor Miles Walker for his permission to have Fig. 82 reproduced, and to the Weston Electrical Instrument Company for Fig. 60. His sincere thanks are also due to Mr F. J. Dykes, M.A., Fellow and Tutor of Trinity College, for much useful criticism and for kindly revising the proofs, and to Mr J. B. Peace, M.A., the University Printer, for suggesting many improvements in the original draft.

C. G. L.

CAMBRIDGE,
October, 1921.

CONTENTS

ALTERNATING CURRENTS

1. **Production of E.M.F.** The simplest possible form of dynamo for producing an alternating E.M.F. consists of an ordinary two-pole one with a single wire fixed on the rotating armature core and attached at its ends to two rings, or the exactly equivalent form in which the poles rotate and the wire is at rest. It is usually more convenient to depict the arrangement on a straight line time or distance base instead of the actual form, as is shown in Fig. 1. The distance a occupied by a single polar flux is called the pole pitch; it may also be reckoned in radians round the arc of the armature core and will then be π. The pole will produce a definite distribution of flux in the air-gap the shape of which distribution

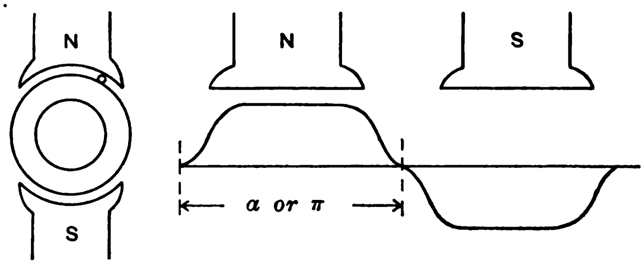


Fig. 1

can be modified by such means as altering the form of the polar tips or varying the thickness of the gap from point to point, but must be taken to be the same for each pole. The curve connecting the induction in the air-gap and the distance along it is called the Field Form curve and may have a shape such as that shown in the figure, so that at any point along the circumference, preferably reckoned from the point of zero flux, there will then be a definite induction B, which usually has a single maximum denoted by B_m.

One of the quantities specifying the E.M.F. is its frequency f, that is to say the number of times it goes through a complete cycle in one second; the wire will move, relative to the poles, at a certain rate which is such that it passes across the pair of poles in the time T seconds, called the periodic time, and it follows that $f.T = 1$. Further, we may suppose that instead of a single wire we

have a set of wires, w in number, all in series and all occupying the same place. If we denote by l the length of any wire in the flux and parallel to the axis of the armature, we know from first principles that the E.M.F. generated in the set at any point is given by $e = wlvB$, where v is its velocity in cm. per second. But we have $2a = vT$ and hence $v = 2af$; so that the instantaneous E.M.F. in the set of wires has the value $e = 2walfB \times 10^{-8}$ volts. We see also that the shape of the volt time curve is the same as that of the field form curve.

There is one derived E.M.F. that we can always determine, namely the mean or average E.M.F. $[e]$; this is given by

$$[e] = 2walf\ [\text{average } B].$$

It follows that if ϕ denote the total flux emerging from a pole, so that $\phi = al$ [average B], we can derive the average E.M.F. from the total polar flux by the formula $[e] = 2wf\phi$ irrespective of the shape of the field form curve.

The actual practical measure of the E.M.F. however is its virtual or root-mean-square value, and this is given by

$$E = 2walf\ [\text{average } B^2]^{\frac{1}{2}}.$$

Hence the relation between the required virtual E.M.F. and the easily found mean is

$$\frac{E}{[e]} = \frac{(\text{average } B^2)^{\frac{1}{2}}}{\text{average } B},$$

so that E depends on the shape of the above curve, and cannot be determined unless we know it.

For any given field form curve the ratio (average $B^2)^{\frac{1}{2}}$ to (average B) is called the field form factor and is usually denoted by k. It follows that the virtual E.M.F. is given by the equation

$$E = 2kwf\phi \times 10^{-8} \text{ volts},$$

where k is a factor depending on the nature of the field form curve which may have very diverse shapes.

The formula given above involves f, the frequency, that is, the number of times per second the wire cuts a complete cycle of flux. If the electrical conditions require a definite frequency f, while the mechanical conditions impose a dynamo speed of N R.P.S., we must have the relation $f = NP$, where P denotes the number of pairs of poles. That is to say, the number of pairs of poles provided must be such as to satisfy this relation.

As far as actual electrical questions are concerned we can very often use the single pair of poles for elucidation and this saves much circumlocution. When we have many pairs of poles instead of one, it will be seen that we can replace the single set of w wires appropriate to the single pair by any equivalent distribution; for example, we could use P sets spaced out symmetrically under similar poles with the original direction of winding provided we give each new set w/P turns, or we can also use $2P$ sets symmetrically placed under all the $2P$ poles, each set with $w/2P$ wires, provided we connect up alternate sets so that the E.M.F.'s produced under each pole add up, that is to say, so that alternate sets are connected in opposite ways. The methods of arranging this connection are very numerous, and each has its special advantage for special purposes; the reader should consult works on design for these details.

Although we have supposed that the wires form but a single circuit other arrangements are possible. We can wind the armature with two sets of wires in parallel if desired, when w must denote the actual number in series, not the total number; a very common form of this condition is provided by attaching two collecting rings to opposite points of a direct current dynamo armature from which alternating currents can then be taken. In this case the necessary relation between the two pressures is that the maximum of the alternate current pressure must be equal to the steady value of the direct current one: what the virtual value is will depend on various questions, such as the field form of the machine. This gives a very convenient source of alternate currents especially when the armature is being driven as a direct current motor at constant P.D., since the speed and hence the periodicity can be so easily altered without affecting the virtual value, a useful property in many tests.

2. Sinoidal Field Form. It would clearly be an advantage if we could refer all our considerations to some standard field form which would avoid the difficulties due to variations in the constant k, and for this purpose the best curve to take is a sine curve. The actual sine to assume will depend on circumstances; it may be one accounting for the same total flux, when we have still our equation $E = 2kwf\phi \times 10^{-8}$, but can now determine the value of k. For with a sine field form curve, which has a definite maximum, namely B_m, we know that the (mean $B^2)^{\frac{1}{2}}$ is $B_m/\sqrt{2}$,

while the mean B is $\frac{2}{\pi} B_m$, and the factor k is $\pi/2\sqrt{2}$ or $1{\cdot}1$ very nearly; hence with such a field form the virtual E.M.F. is given by

$$E = 2{\cdot}2wf\phi \times 10^{-8}$$

for the same form of winding assumed as before, namely with all the wires symmetrically placed relative to the poles, or "concentrated." A very important property of this field form is that not only will a wire have the same E.M.F. curve whatever its position but also that wires no longer concentrated but placed anywhere will have the same shaped E.M.F. curve, namely a sine one. Hence the summation of the E.M.F.'s produced in any wires can now be represented by means of vectorial addition, which is impossible for any other shaped field form curve.

3. Forms of Winding. The first question affecting the value of the E.M.F. with such a field form is the "pitch" or the distance between any wire and its return circuits; if these are so arranged that they bridge symmetrically from pole to pole, that is to say, if one be at mid-pole so is its return, the arrangement is called a Full Pitch winding. But for some purposes, constructional or electrical, there is an advantage in having the return wire not in exactly the same place opposite a south pole as its fellow is opposite the north. The latter being at mid-pole, the former may be a little behind that position so that the winding pitch is less than the polar pitch; this winding is called a Short Coil winding. Again, we may deal with the question from another point of view, whether the several wires are all at similar points at the same time, so that each pole has a single bunch of wires in front of it, or whether the wires occupy a finite part of the circumference of the core opposite each pole. The former is what we have taken hitherto, being the Concentrated Winding. In general, however, the core has slots in it with a definite number opposite each pole, and in symmetrical or full pitch windings it is of course necessary that the number of slots round the periphery of the core should be divisible by the number of poles. Although the armature core is slotted uniformly all round, it is not essential to place a packet of wires in every slot. If the dynamo be a "monophase" machine with but one armature on the core, we would only use some of the slots as a rule, but with machines having more than one armature, the "polyphase" dynamos, the slots left vacant are used for those other armatures;

thus in a "three-phase" dynamo which has three similar and distinct armatures on the same core we could allot one-third of the slots in the polar pitch to each armature or "phase" and thus fill up all the slots. When the packets of wires are thus spread out along the core we will call the arrangement a Dispersed Winding. The limiting form of this would be the type of winding in which a completely overspun part of the core was used as a coil corresponding to what is very nearly attained in the armature of a direct current machine; this is called a Distributed Winding. The two latter forms might be used with either the full pitch or short coil method of connection.

4. Breadth Coefficient. We will now deal with the full pitch winding and will first consider what is the result of "dispersing" the wires so that they are no longer practically all in similar positions at the same instant. In order to find what alteration in the virtual value of the E.M.F. is produced by such dispersal, we must first find the maximum E.M.F. due to any arrangement; for since the sum of any number of sines of the same frequency is still a sine, the ratio between the maximum of the original and that of the new configurations will be the same as that of the corresponding virtual values. In other words we can now write the E.M.F. equation in the form

$$E = 2{\cdot}2Kwf\phi \times 10^{-8},$$

where K is another constant depending solely on the method in which we distribute the wires rather than leaving them in single packets symmetrically under the poles; the factor K is called the Breadth Coefficient.

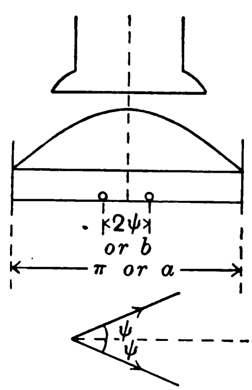


Fig. 2

As an example, suppose we have originally two wires in a packet; at the instant of maximum E.M.F. they will be just under the middle of a pole. Now let them be separated a certain distance apart, Fig. 22; in order that they may be again producing the maximum E.M.F. they must be symmetrically placed about the centre line. For convenience it is best to specify their dispersion by the angle 2ψ they occupy relative to the polar angle π, or alternatively by the distance b

between them relative to the polar pitch a so that

$$\pi : a :: 2\psi : b \quad \text{or} \quad \psi = \frac{\pi b}{2a}.$$

It can be seen that if e_0 was the maximum E.M.F. in a single wire in the original concentrated form, the E.M.F. per wire at the moment of maximum E.M.F. in the dispersed form is $e_0 \cos \psi$ so that K is here $\cos \psi$.

It follows that if we have s pairs of packets subtending angles $\psi_1, \psi_2, \psi_3 \ldots$ etc., provided that the total number of wires in series at a moment is the same as in the concentrated winding,

$$s\,K = \cos \psi_1 + \cos \psi_2 + \cos \psi_3 \ldots.$$

In general the angles differ by a constant amount since windings are usually placed in evenly spaced slots cut in the armature core.

Suppose we have a set of such evenly spaced slots under the pole and that the total number in the polar pitch is S, and let each slot take up the angle 2β where π is the polar pitch, so that $2\beta S = \pi$. Suppose further that instead of s concentrated wires we use s of the total polar slots S to form a dispersed winding; at the moment of maximum E.M.F. they will be as shown in Fig. 3. Since each maximum E.M.F. is given by the normal concentrated E.M.F. e_0 of a wire multiplied into the cosine of the angle made with the line of symmetry, the E.M.F. due to the s slots each with a single wire instead of the s wires in a single slot will be given by

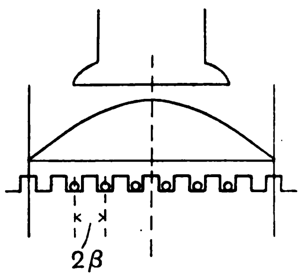


Fig. 3

$$e = 2e_0 \{\cos \beta + \cos 3\beta + \cos 5\beta + \ldots \cos (s-1)\beta\}.$$

Multiply each side by $\sin \beta$ and evaluate the products on the right-hand side, it will be seen that they cancel term by term and leave as a result $\frac{1}{2} \sin s\beta$. It follows that the E.M.F. due to the s slots will be

$$e = e_0 \frac{\sin s\beta}{\sin \beta}.$$

But the "concentrated" value of the E.M.F. would have been se_0, hence the value of K is

$$\frac{1}{s} \cdot \frac{\sin s\beta}{\sin \beta}.$$

It is sometimes convenient to express this in terms of the slots per pole pitch S, and since we have $2\beta S = \pi$ it follows that

$$K = \frac{\sin \frac{s}{S}\frac{\pi}{2}}{s \sin \frac{1}{S}\frac{\pi}{2}}.$$

This will be seen to have an important bearing later on.

The formula just arrived at has a very simple geometrical meaning; it expresses the sum of a specified number of vectors placed in succession at a common angle, and can thus be derived as follows. On any base draw a semicircle, and at the centre set off the given angle 2β as shown in Fig. 4. This is an integral fraction of π, so that the vector $O1$ can be stepped round as many times as there are slots in the polar pitch, and will close up on the diameter at the last step. If the ends of the steps be joined to O it follows that if $O1$ be the maximum E.M.F. per wire in a single slot, $O2$ will be the E.M.F. if we use two slots, $O3$ for three and so on; with a monophase or ordinary machine we can use as many as we choose. The figure is drawn for $\beta = 15°$, corresponding to an armature with 6 slots to the pole pitch. If we take the coil slots as 4, the breadth coefficient will be $O4/4.O1$. In such cases as mentioned above where we wind other armatures on the same core, s must be less than S; thus a "two-phase" armature would involve half the slots being used for each armature, so that the breadth coefficient will be that

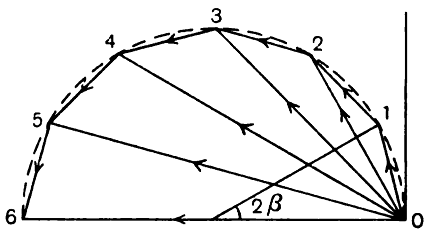


Fig. 4

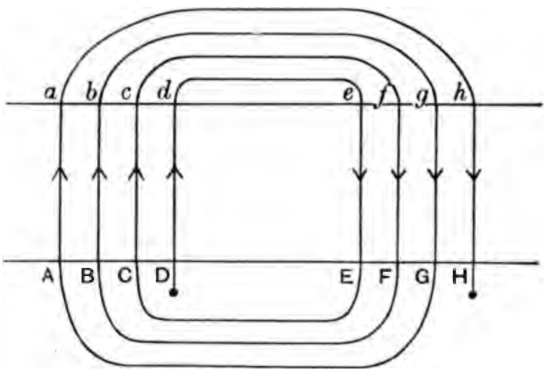


Fig. 5

for three slots, or $O3/3.O1$. If we have a "three-phase" one, we would use two slots per armature giving $O2/2.O1$ for the coefficient.

It may be noted that with a full pitch winding the actual method in which the wires are assembled in series is quite a secondary matter and can be settled for reasons of constructional simplicity. Thus if the four wires under each pole in Fig. 5 be full pitch, it is quite indifferent how we join them up, provided the arrow heads are concurrent; in the connection made we might connect in the way shown in Fig. 5 or indeed in any convenient manner; the coil for all electrical purposes is full pitch for any such connection.

We will now consider the fully distributed winding, that is to say one which consists of a coil closely wound over a more or less limited part of the core, like a portion of a direct current armature, Fig. 6. From symmetry the instant of maximum E.M.F. for the coil is as shown, and we will specify its width by the angle 2ψ; let e_0 be the maximum E.M.F. per wire in the concentrated case, namely at mid-pole. Since the E.M.F. in a wire follows the field form curve, that in a wire at the angle θ must be $e_0 \sin \theta$. Suppose the winding is such that over 2ψ we have σ wires per radian, then in a slice $d\theta$ of the coil the E.M.F. must be $\sigma e_0 \sin \theta \, d\theta$, so that the E.M.F. for the whole coil at the instant of maximum as figured will be

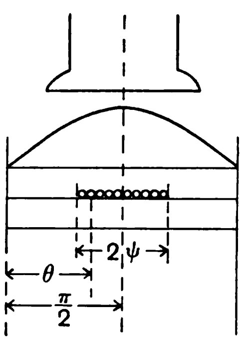


Fig. 6

$$\sigma e_0 \int_{\pi/2-\psi}^{\pi/2+\psi} \sin \theta \, d\theta.$$

This leads to $2\sigma e_0 \sin \psi$. If we denote the total turns in the coil by m, we have $m = 2\psi\sigma$ so that the ratio of "E.M.F. distributed" to "E.M.F. concentrated," that is K, is here $\sin \psi/\psi$. A special case is when the distribution is complete as in a direct current machine: the value of K is then $\dfrac{\sin \pi/2}{\pi/2}$ or $2/\pi$.

The distributed breadth coefficient like the dispersed one has a simple geometrical meaning, for it will be apparent that it is merely the ratio between the chord and the arc of the angle 2ψ.

5. Short Chord Winding. So far we have supposed that every wire under a north pole has its fellow under the south pole

in exactly the same relative place, but it is quite common, as said above, to use coils which span less than the full pitch, or "short coil" windings. Let the coil (Fig. 7) span less than the full pitch π by the angle 2η, then at the moment of maximum E.M.F. from symmetry it must be situated as shown with the wires equidistant from the mid-poles; the wires are in series with one another as indicated by the cross and dot in the circles; it follows from the figure that the maximum E.M.F. will be less than before by the factor $\cos \eta$. Since everything is sinoidal, this holds throughout; hence there is an additional factor to consider with short coils, the "coil-span factor" or $\cos \eta$, in addition to either the slot factor for dispersed windings or the factor for fully distributed ones. This arrangement has apparently no particular advantage in respect to a pure sine field form, as it only results in a further diminution of the E.M.F. produced and is hence apparently of no importance, but we shall see that it leads to some very useful possibilities when we deal with field forms other than sinoidal ones.

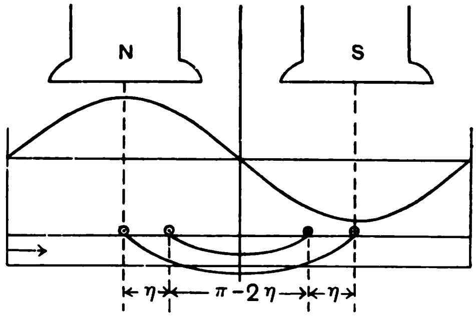


Fig. 7

With full pitch windings symmetry of the successive coils relative to successive poles is implied in the term, but this is not necessarily so with short coil windings, as each group of coils can be slightly displaced relative to the mid-line of successive pairs of poles. Suppose we have a fully concentrated winding, then while the coils opposite a

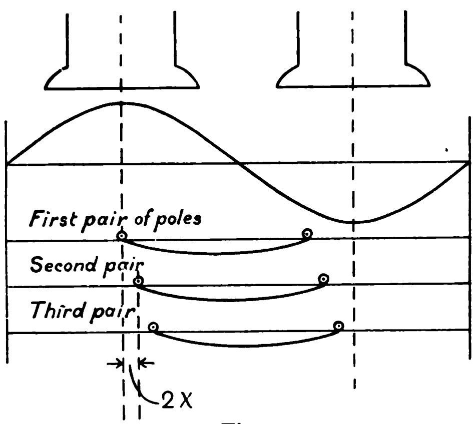


Fig. 8

specified pair of poles are as shown in the upper line of Fig. 8 those opposite the next pair may be a little ahead of the first, those opposite the third pole pair again a little ahead, and so on as figured. Indeed we may suppose that we have m groups of coils each displaced relative to the previous one by the angle 2χ. This will as a rule involve some want of symmetry such as an empty slot or slots in the periphery and means that the number of slots need no longer be integral with respect to the number of poles; this arrangement again offers additional advantages with non-sine field forms, but involves another factor in the breadth coefficient. It will be seen that the condition of m groups each advancing by 2χ relative to the successive field forms is exactly the same problem as successive slots in the dispersed winding each advancing the angle 2β under a single pole, and hence involves a factor of exactly the same form, namely

$$\frac{\sin m\chi}{m \sin \chi}.$$

Each group need not be concentrated, it may be dispersed and hence will be affected by the appropriate factor. It follows that in our expression for the E.M.F. $E = 2{\cdot}2Kwf\phi \times 10^{-8}$; when we have short coil, non-integral groups, and dispersed windings the value of K, then often called the "winding coefficient," will be

$$K = \cos\eta \frac{\sin m\chi}{m \sin\chi} \frac{\sin s\beta}{s \sin\beta}.$$

With full pitch windings, on the other hand, $\cos\eta$ becomes unity, and the factor depending on the non-integral arrangement of the groups also becomes unity, hence for such windings we have

$$K = \frac{\sin s\beta}{s \sin\beta}.$$

Finally for completely distributed ones we have

$$K = \frac{\sin\psi}{\psi}.$$

It follows that, for a sine field form, we can fully determine the value of the E.M.F. for any assigned form of winding.

In nearly all ordinary windings the three quantities η, χ, β are not independent, but are related to the slot pitch. Let us consider an armature with P pairs of poles, and let us select a specific wire that is just at the middle of a pole; since the pitch of the wires is no longer that of the poles it is possible that on proceeding round

we do not find another wire in a similar situation, but on the other hand the pitch and number of poles may be such that we do. Suppose that in our progress right round the armature we find that we encounter a similarly situated wire A times, then if m is the number of groups on the armature we must have $P = Am$, the sole condition being that A must be an integer. If it be unity, a common condition, it means that we have as many groups as there are pairs of poles. Now let Σ denote the total number of slots round the whole circumference, and as before, S the number, either integral or fractional, in the polar pitch, of which either the whole or the smaller number s are to be filled with coils. Since there are P pairs of poles we must have $P\,2S = \Sigma$, so that since $P = Am$, we have $\Sigma/A = 2mS$. If we are going to use a total number of slots Σ which is not exactly divisible by A, S becomes fractional, but we must necessarily have an integral number of slots in which to wind any given coil, so that the slots occupied by them will be given by $2mS_0$, where S_0 is an integer near to S. These two quantities $2mS$ and $2mS_0$ will themselves differ by some integer, and the usual one to take is unity. It follows that the relation between the two pitches is given by

$$2mS = 2mS_0 + 1.$$

We will only consider here the arrangement which gives the greatest possible grouping, namely when A is unity or $m = P$. The relation between the two pitches then becomes

$$S = S_0 + \frac{1}{2m} = S_0 + \frac{1}{2P}.$$

Since the difference in angular pitch between the successive slots is the angle 2β it follows that the fractional pitch multiplied by that angle is the pole pitch or $S\,2\beta = \pi$. So that we now have

$$S_0\,2\beta = \pi - \beta/m \text{ or } \pi - \beta/P,$$

that is, the pitch of each short coil will be $\pi - \beta/P$, from which it follows that the appropriate value of the angle η for the coil pitch constant in the general formula is $\eta = \beta/2P$.

In a similar way we can see that the group angle χ is likewise tied on to β. For 2χ is such an angle as to express the amount by which we advance from group to group; with m such advances we have a total advance of one slot, namely 2β. Hence we must have

$$m\,2\chi = 2\beta \text{ or } 2\chi = 2\beta/m = 2\beta/P.$$

Thus, in the practical case of a single extra unwound slot the three angles η, β, and χ are related as above, and the resultant total breadth coefficient is easily found in terms of the single angle β.

As an example consider a three-phase machine with three slots per pole per phase, so that the slots per pole pitch with full pitch winding would be 9. Also let it be four pole, then the total number of slots for that condition would be 36; but we are taking one more, that is 37. It follows that we will have $S_0 = 9$ and $S = 9\frac{1}{4}$. The angle 2β is then given by $\pi/9\frac{1}{4}$ or is 19° 27′ 40″. Draw a semicircle, Fig. 9,

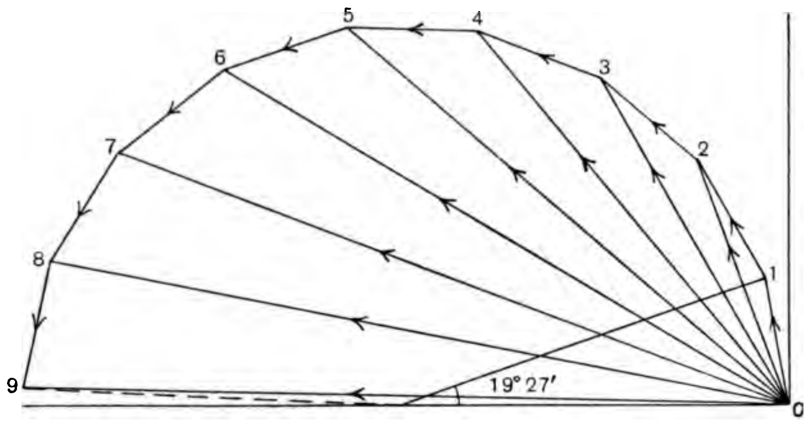


Fig. 9

and set off this angle as indicated. It can readily be done by noting that $2 \sin \beta$ is 0·6658, so that a radius can be struck from O cutting the circle in the point 1; step round nine times and we have set off the ends of the nine vectors for the slots; the last one falls short of the whole 180° by the amount 4° 51′, which is of course the value of β/P since $P = 2$. It follows that the coil pitch falls short of 180° by the angle 4° 51′, and thus the coil pitch coefficient of p. 9 is cos 2° 26′ or 0·996. The group coefficient can be readily found: there are two pairs of poles, and hence with a single slot unwound there are also two groups, so that the value of the group advance angle χ is $2\beta/P$ or β. The group factor being $\dfrac{\sin m\chi}{m \sin \chi}$ it reduces to $\dfrac{\sin 2\beta}{2 \sin \beta}$ or simply $\cos \beta$, and since β is 9° 43′ it has the value 0·986. These two coefficients are independent of the number of slots per pole that are utilized. Let us suppose that we are going to wind the core as a three-phase machine so

that three of the polar slots will be required for each winding. The last coefficient, that for dispersal, will then be given by the ratio of the length $\overline{O3}$ to three times the length of $\overline{O1}$, that is to say will be 0·976. The product of these three factors is the complete winding factor for the machine. In the same way if all the slots were used for a monophase machine, the distribution coefficient would be the ratio of $\overline{O9}$ to nine times the length of $\overline{O1}$, that is to say, 0·664.

6. Non-sine Field Form. The combination of a non-sine field form and a distributed winding cannot be solved in any simple mathematical manner. It must be dealt with by a graphical method, and the simplest modification is that due to Professor

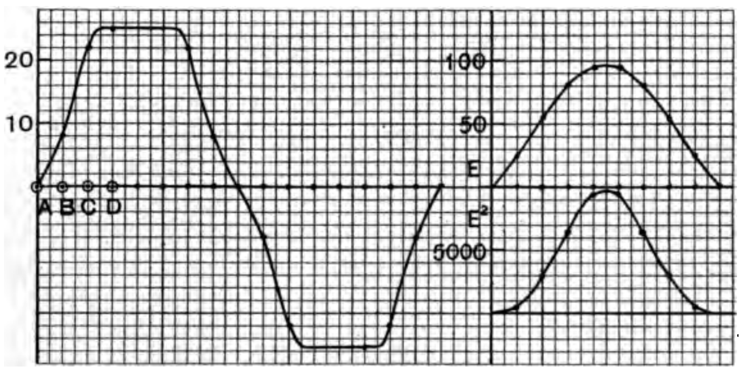


Fig. 10

Miles Walker. Suppose we have a winding in which the element consists of four equally spaced sets of wires as shown at *A*, *B*, *C*, *D* in Fig. 10. The E.M.F. in a single wire being $e = 2lwfB$, it is merely a question of knowing the number of wires in each packet and the other constants to enable us to alter the scale of the field form curve so that its ordinates express the instantaneous values of the E.M.F. in each packet; the method is then best seen by an example.

Let the E.M.F. time curve derived from the field form curve be as in Fig. 10, where the ordinate is the E.M.F. due to the set of wires in a slot; in this case the E.M.F. is taken per pole for simplicity. Mark off as shown the position of the four wires, preferably with one of them at the zero of the curve. In the column headed "ordinates" set down the value of the four E.M.F.'s due to the four

slots and add them up. If we imagine the set of wires to be moved forward the distance between two adjacent ones, we shall cut out the E.M.F. of the extreme left slot and add in that at the corresponding position to the right. The new E.M.F. will therefore be given by subtracting the first ordinate and adding the latter. Draw

−	+	Σ	Ordinates	(Ord.)^2
...	...	...	0	...
...	...	...	8	...
...	...	...	22	...
...	...	...	25	...
...	...	...	55	3025
0	25	25	80	6400
8	25	17	97	9409
22	22	0	97	9409
25	8	−17	80	6400
25	0	−25	55	3025
25	− 8	−33	22	484
22	−22	−44	−22	484
8	−25	−33	−55	3025

up two columns marked − and + such that the first gives the successive values of the ordinate left behind, whilst the other gives the value of that included at each step. Take the sum of these as shown in the column Σ and add the successive values of Σ to the initial and subsequent values in the ordinates column. Proceed until you arrive at a negative value exactly equal to the initial one with which you started. It follows that if we plot these resulting numbers to a time base, they will give very approximately the form of the E.M.F. curve for the four packets in series. To determine the virtual value of the E.M.F., square each ordinate and plot the resulting curve as shown, determine its mean height and take the square root of that quantity. In the given figure this amounts to about 68 volts, so that an eight-pole machine will give about 550 volts.

7. Harmonics. It has been seen that a sinoidal variation offers many advantages in simplifying calculations, and we must now consider whether we can use this to elucidate any arbitrary field form. It can be stated in terms of Fourier's Theorem that any periodic curve can be built up from a set of sine curves provided three matters can be arranged for: (1) that the sine curves building up the other curve consist of a main one of the same frequency as that of the given curve, which sine is called the Fundamental, together with an indefinite number of other sine curves called the

Harmonics, whose frequencies or "orders" are 2, 3, 4 ... n times as fast as the fundamental, (2) that the amplitudes or maxima of these various sines are suitably chosen, (3) that the times of zero passage of each relative to that of the fundamental are known. In other words, if y denotes the value of any ordinate of the given

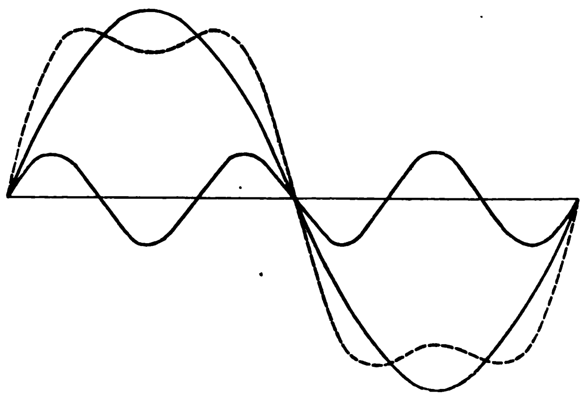


Fig. 11

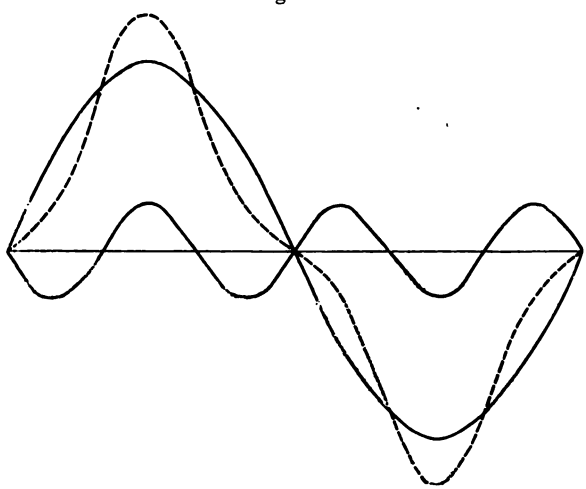


Fig. 12

curve and if we use the angle θ for the independent variable, we can write

$$y = A_1 \sin \theta + A_2 \sin 2 (\theta - \epsilon_2) + A_3 \sin 3 (\theta - \epsilon_3) + \dots,$$

where the A's are the maxima and the ϵ's are the angles of zero passage. No complete proof will be given of the proposition as this

is unnecessary for our purpose: we will illustrate it by considering a fundamental and the third harmonic. If we suppose that they start off co-phased at zero we get the sum in the form of the somewhat flat topped curve, Fig. 11, while if they start off at

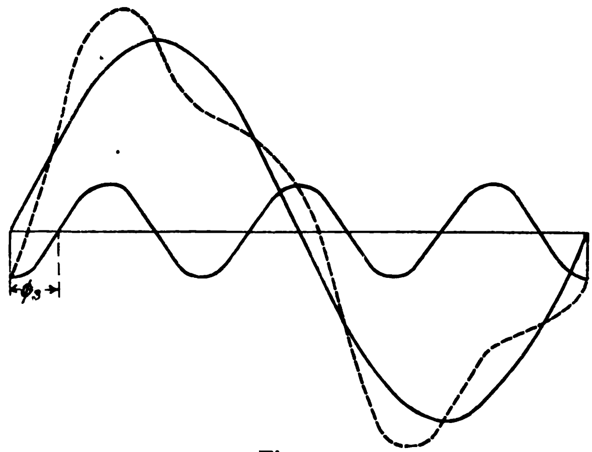


Fig. 13

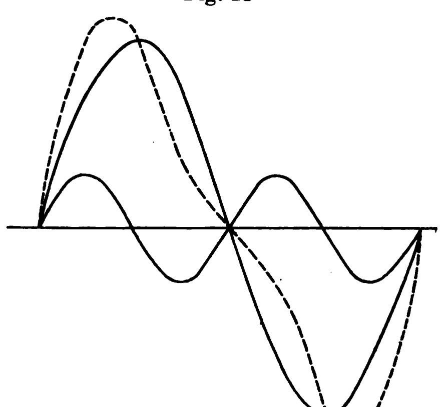


Fig. 14

opposite phase we get the pointed one, Fig. 12. If we start with any other initial phase relation we get a curve like Fig. 13, in which the maximum is non-symmetrical with respect to the centre line. It will readily be perceived that by a suitable choice of a great number of harmonics, each with the proper values for its "A" and "ϵ," we can probably approximate as nearly as we please to

any given curve. Fortunately in nearly all practical problems the number of components necessary to reproduce a given curve is comparatively small; and the fundamental is by far the most important member. But most electrical quantities permit of a further simplification. Consider a fundamental and the second harmonic; the result of adding these is as seen in Fig. 14 and involves of necessity that the maximum is ahead of the centre line in one half of the curve, and behind it in the other: such a state of affairs cannot exist in any ordinary electrical or magnetic problem, as it involves the positive and negative waves being dissimilar in respect to the time of their maximum, which is impossible unless it is purposely brought about in some special way. It follows therefore that we need only consider the odd harmonics, namely the fundamental and those of orders 3, 5, 7, etc., so that we can write

$$y = \Sigma_1^q A_q \sin q\,(\theta - \epsilon_q),$$

where q is any odd number from 1 to infinity. In a great number of important cases a still further simplification is possible, since many practical curves are symmetrical about the maximum ordinate, the falling half of the wave being a mirror image of the rising half. When this is so it will be seen that the only possible relation of the component sines is one of initial co-phase or anti-phase, so that we then have

$$y = \Sigma_1^q A_q \sin q\theta.$$

8. Space and Time Harmonics. So far nothing has been said as to what the angle θ represents, it is merely the independent variable, but two important applications must carefully be kept distinct. We may be desirous of analysing a quantity which is varying with the time, and if this be so, we must remember that for a periodic time T, or frequency f, we have

$$\theta = 2\pi t/T = 2\pi ft.$$

But another very important set of quantities are those which have a periodic arrangement in space, such as our field form curves. Here also we wish to break the curve up into a set of sines, and we generally select for our zero angle the point where the ordinate of the curve is zero; if we denote by x any distance along the air-gap reckoned from that point, by ξ the corresponding angle in the Fourier series, and by a the pitch of the curve, we have $\xi = \pi x/a$. The distinction between the space harmonics and the time harmonics must be borne in mind very carefully, since in some

apparatus we have both a space distribution of a quantity and its time variation to consider simultaneously, and hence each variation may have quite different harmonics both as to size and frequency: to emphasize this distinction θ will be kept for the time angle and ξ for the space angle.

As an example of the coexistence of both space and time variations, consider a coil through which the flux-turns at any instant are ψ; let it be capable of motion and at the same time let the flux be independently changing. We know that the E.M.F. demanded by it must always be $e = \frac{d}{dt}\psi$. But we can write this in the form

$$e = \frac{\partial\psi}{\partial x}\frac{dx}{dt} + \frac{\partial\psi}{\partial t},$$

showing that we must then bear in mind two E.M.F.'s, one due to cutting the actual flux at any moment by physical motion, the other due to the change of flux in the coil if that be supposed at any instant to be momentarily at rest. These separate aspects of the flux-change are familiar; in the ordinary direct current machine the flux is steady with time, but motion takes place, so that we are dealing with the first part of the equation or

$$e = \frac{\partial\psi}{\partial x}\frac{dx}{dt},$$

while in an ordinary dynamo $\partial\psi/\partial x$ is a constant. On the other hand in a transformer we are dealing with stationary iron masses, but the flux varies with time, so that we then use the other half of the equation or $e = \frac{\partial\psi}{\partial t}$. In some machines we have both events occurring simultaneously; for example, imagine an armature to be supplied with an alternating current, and let the field magnet be similarly supplied at the same frequency. We then have the complete case; any circuit on the armature will have in it two alternating E.M.F.'s of the same period, a "rotational" one due to cutting the instantaneous flux at the speed of rotation, and a "transformer" one of the same periodicity due to the direct rate of change of the coil-flux with time. The most important circuit on the armature would be that of the armature between the brushes, and hence these two E.M.F.'s will be operative there, a rotational one produced exactly like that of an ordinary motor or dynamo, but periodic and not constant as in the direct current motor, and a transformer one whose value depends on the ratio

of transformation between the simply wound field and the distributively wound armature. It is of importance to note that since the rotational E.M.F. is due to the term $\frac{\partial\psi}{\partial x}\frac{dx}{dt}$, while the other E.M.F. is due to $\frac{\partial\psi}{\partial t}$, the two E.M.F.'s must be in time-quadrature with one another. The importance of keeping the two harmonic distributions distinct is now very clear, the rotational E.M.F. will depend on the way the flux changes with the position of the coil, the transformer E.M.F. depends solely on the actual flux caught at the instant. Hence as the shapes of the time curve and space curve may be very different, it will be seen that in order that the total E.M.F. may be sine-shaped, both its components must be so also, and hence both the time variation and space distribution of the flux must be sinoidal.

There are many mathematical or graphical methods by which it is possible to effect a complete analysis of a given time or space curve into its constituent harmonics, both in respect to amplitude and phase, but it is very rarely that this analysis is actually required. We may distinguish three degrees in our requirements, (1) a knowledge of the fact that certain harmonics are present or absent, (2) a knowledge of the amplitudes only, (3) a complete knowledge of the curve's components. As regards the first, the absence or presence of certain harmonics can often be predicated from inspection of the data or from simple geometrical or physical reasons: cases will be given later on. As regards the second, this is required for calculating the virtual value of the compound curve from the amplitudes of the constituent harmonics. Consider any two of the harmonics whose orders are m and n, so that they are given by $A_m \sin m\theta$ and $A_n \sin n\theta$; the square of the whole quantity will then involve the sum of the squares of each harmonic and of all such products as $A_m A_n \sin m\theta \sin n\theta$ taken over the periodic time of the curve, T, or the corresponding angle 2π. As regards the product, this can be written in the form

$$\frac{A_m A_n}{2}\{\cos(m-n)\,\theta - \cos(m+n)\,\theta\}.$$

But since m and n are both integers, so are $(m-n)$ and $(m+n)$, and it follows that the mean value of this product over the complete period 2π is zero, so that the products contribute nothing to the result. On the other hand the mean value of any one of the

squares is $\frac{1}{2}A^2$, so that the mean of the squares of the quantity is equal to the sum of the means of the squares of the harmonics; in other words, the virtual value of the quantity is the square root of the sum of the squares of the virtual value of each harmonic.

The full knowledge of the curve is required in space distributions especially when we are dealing with their magnetic effects, since these depend essentially on the shape of the curve expressing the distribution; fortunately in such cases it is nearly always sufficient to calculate only the value of the fundamental. For some problems a partial knowledge of the curve suffices, such as a knowledge of its maximum and this can sometimes be found by an indirect method. For example, the maximum of the induction in a transformer can be derived from the measured value of the mean E.M.F. and that of the virtual as described in *Notes on Magnetism*, p. 86. This determination is independent of the form of the curve and is all that is required when we are dealing with the iron losses in relation to the induction.

Although it is not proposed to show how the amplitudes of the several harmonics in the relation $y = \sum_{0}^{\infty} A_q \sin q\theta$ can be found, yet it is useful to be able to find them for several standard cases. Suppose we have to represent the quantity shown in Fig. 15 by such a series, where α, β and γ are the three sections (in radians) of the quarter period $\pi/2$ of the function, and Y is the maximum, it can readily be shown that the value of any coefficient is given by

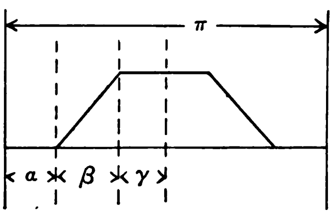


Fig. 15

$$A_q = \frac{8Y}{q^2\pi\beta} \sin \frac{q}{2}\beta \cos \frac{q}{2}(2\alpha + \beta).$$

From this we can get close approximations to many usual types of curve. Thus if we make α and β both zero, we get a uniform quantity over the half period; with β zero we get a similar quantity over part of the period; with α zero we have a very usual form, a trapezium; while with α and γ zero and β equal to $\pi/2$ we have a peaked triangular wave. This formula then enables us to find approximately the harmonics in most of the usual wave forms found in practice.

A deduction of some interest and importance is the following: the function includes a uniformly sloping part over the angle β,

and it will be noticed that the size of the harmonics diminishes as the square of their order. Let the part β be absent so that there is a rectangular function. Then $\sin \frac{q}{2} \beta / \beta$ assumes the form O/O or with the usual operation becomes $\dfrac{\left(\frac{q}{2} \cos \frac{q}{2} \beta\right)}{1}$ $[\beta = 0]$, that is $q/2$. Hence the harmonics now fall off far less rapidly, as their order and not as its square. Thus if a field form curve has a slope in it, such as is approximately produced by the polar fringing, the harmonics in its analysis will fall off far more rapidly than with a square shaped curve.

9. Breadth Coefficients of Harmonics. We must now consider how the harmonics are affected by the form of winding employed, in other words, their breadth coefficients. With a full pitch perfectly concentrated winding, the E.M.F. curve must exactly reproduce the field form, but it is quite different with the other windings. With a sine field form, however the winding was arranged, whether dispersed, distributed or even when short coil, the resulting E.M.F.'s were bound to be sinoidal but with differing maxima. When harmonics are present each will have its appropriate breadth coefficient depending on the nature of the winding used, and we must now consider these. Let us first take the fully distributed winding with a third harmonic in the field given by $B_3 \sin 3\xi$, and let a distributed coil like that in Fig. 16 be in action. Everything established for the fundamental sine considered on p. 8 will hold true and we can derive the equation for the maximum E.M.F. in the form

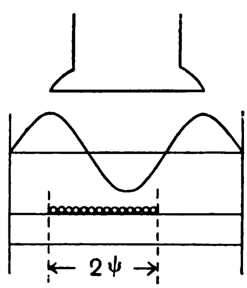


Fig. 16

$$\sigma e_0 \int_{\pi/2-\psi}^{\pi/2+\psi} \sin 3\xi \, a\xi, \text{ which gives } \frac{2\sigma e_0}{3} \sin 3\psi.$$

But the E.M.F. due to the coil of $2\psi\sigma$ wires when concentrated would have been $2\psi\sigma\epsilon_0$, so that the value of the breadth coefficient for the third harmonic is $\dfrac{\sin 3\psi}{3\psi}$. In exactly the same way for the qth harmonic we have $\dfrac{\sin q\psi}{q\psi}$. These coefficients evidently fall off rapidly with the order and hence the relative importance of the

harmonics in the flux curve is greatly diminished in the resulting E.M.F. curve. In fact it will be seen that any desired harmonic can be abolished, since any one for which $\sin q\psi$ is zero cannot appear in the pressure curve. Thus in order that no third harmonic may be there, a very important condition in many cases, we must have $3\psi = \pi, 2\pi$, etc. But we also have $2\psi/\pi = b/a$; hence the condition is $b/a = 2/3$ or the coil must occupy two-thirds of the polar face. This of necessity also involves the disappearance of 9th, 27th, etc. harmonics. Similarly the 5th harmonic will be absent if $5\psi = \pi$, or if $b/a = 2/5$ or $4/5$. This is an example of how we can get important information relative to the presence or absence of the harmonics from inspection of the form of the winding only. The dispersed winding leads to somewhat similar results. We saw on p. 6 that in a winding of s wired slots pitched at the angle 2β, where 2β is equal to π divided by the total slots S per pole, the breadth coefficient for a sine field form curve was given by $\frac{1}{s}\frac{\sin s\beta}{\sin \beta}$. A simple substitution shows that the corresponding coefficient for the qth harmonic with the same set of slots is $\frac{1}{s}\frac{\sin qs\beta}{\sin q\beta}$. This is very different from the expression for the distributed winding; it will be noted that both numerator and denominator are harmonic quantities, and hence the denominator will not necessarily fall off indefinitely with increase in the harmonic's order. In fact it may happen that one of the higher order harmonics will appear in the resulting curve as strongly as does the fundamental. For this to occur all that is necessary is that the breadth coefficients shall have the same value, or in terms of the polar pitch slots S we must have

$$\frac{1}{s}\frac{\sin s\beta}{\sin \beta} = \frac{1}{s}\frac{\sin qs\beta}{\sin q\beta} \text{ or } \frac{\sin \frac{1}{S}\frac{s\pi}{2}}{\sin \frac{1}{S}\frac{\pi}{2}} = \frac{\sin \frac{q}{S}\frac{s\pi}{2}}{\sin \frac{q}{S}\frac{\pi}{2}}.$$

It will be seen that this condition is satisfied if $q = 2S \pm 1$, that is to say, there is a definite relation between the number of polar slots and those harmonics which are not relatively diminished by the dispersion of the winding. For example, an armature having six slots per pole will show two harmonics with 11 and 13 times the fundamental frequency which are as little quenched out as is the fundamental, while the same field form with fully distributed

winding would have shown a very small proportion of those harmonics. Of course the identity can be satisfied in an indefinite number of ways; thus the relations

$$q = 4S \pm 1, \quad q = 6S \pm 1, \text{ etc.}$$

will satisfy the same; these give rise to harmonics of the orders 23 and 25, 35 and 37. In general only the first set are of any importance; these unquenched outstanding harmonics are known as "the spacing ripples."

With short coil windings we saw that there was another factor involved, namely the coil-span factor, $\cos \eta$, where 2η is the angle by which the coil-span falls short of, or is in excess of, the full span π. For each harmonic there will be a similar coil-span coefficient which is of course $\cos q\eta$. This gives us another method of eliminating any desired harmonic, namely by properly selecting the angle η. To suppress the qth harmonic, we must have $\cos q\eta = 0$ or $q\eta = \pi/2$ or multiples thereof. Thus to abolish the third we must make η such that $3\eta = \pi/2$ or $2\eta = \pi/3$, so that the coil span is $2\pi/3$. Indeed this solution is self-evident, for any single loop will then just cover the space on the pole that is at any moment occupied by the complete wave-length of the third harmonic, so that it can never produce any nett flux in a coil of that angular breadth; similar results hold for the 5th and others. Hence we can, if we desire, abolish any two harmonics, say the 3rd and 5th; the former can be got rid of by adjusting η as we have just seen, the latter by adjusting β in each of the constituent distributed units.

An advantage of using a pitch non-coincident with the pole pitch is that the harmonics are as largely quenched as with a distributed winding. Practically complete distributional quenching can be secured by having a fractional number of slots per pole, which can be provided by leaving one or two slots unwound. In this case there will be practically no outstanding harmonics, and the spacing ripples cannot exist to any extent. Similar results can be obtained if the pole shoes are skewed from front to back by an amount equal to the pitch of a slot; for the wires at any instant will then occupy all possible positions relative to the field form, so that the actual winding becomes practically a uniformly distributed one, indeed the armature will act as if it were smoothly wound without teeth at all. Other methods are available for producing the same effect such as altering the shape of the polar horns,

etc., but the non-fractional pitch winding is probably the simplest and cheapest method to employ.

10. Tooth Ripples. Hitherto we have supposed that the flux into the armature was, as it were, perfectly "stiff" and did not move in space, but this is not quite true. Consider a toothed armature moving across the polar face; the flux is more or less concentrated in little brushes where the teeth are nearest to the polar face (Fig. 17). Now suppose a tooth A is approaching the polar horn, the flux into it will rapidly grow while that in the tooth B receding from the opposite tip will be rapidly diminishing. Hence the distribution of the flux in space cannot be rigid but will (as it were) sway to and fro. The period of this sway will depend on the number of slots passing under a pole per second; for one complete period of the E.M.F. twice the number of slots per pole must pass across the face. Now the chief factor in the field form is the fundamental for which the induction is given by $B = B_m \sin \theta$; owing to the to and fro sway of the flux, we may roughly represent the effect of the fluctuation by supposing that everywhere the value of the flux cut will vary with the time in such a manner that the maximum, and consequently every other value of B, varies with the periodicity of the sway. In other words we can suppose that the maximum itself varies so that on the steady maximum is superposed a small sine fluctuation of the amount $B_s \sin 2S\theta$; the true induction anywhere is now given by

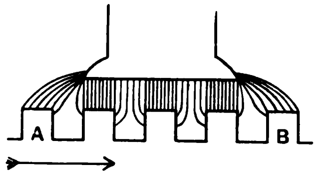


Fig. 17

$$B = (B_m + B_s \sin 2S\theta) \sin \theta.$$

It follows that in addition to the normal E.M.F. due to $\frac{d}{dt} B_m \sin \theta$ there will be superposed another, namely $\frac{d}{dt} B_s \sin 2S\theta \sin \theta$, due to the sway. The latter is equivalent to

$$\frac{B_s}{2} \frac{d}{dt} \{\cos (2S - 1) \theta - \cos (2S + 1) \theta\},$$

so that two new harmonics of the E.M.F. are introduced of periods $(2S - 1)f$ and $(2S + 1)f$; these are the "tooth ripples," and are sometimes of large value. It should be noticed that they coincide

in period with the spacing ripples, and with fractional pitch will be reduced in the same way as are those ripples.

11. **The Oscillograph.** A brief account of a method by which the shape of alternating current pressures and currents can be made visible must be given. There are numerous methods available in laboratory work, most of which involve taking measurements of the desired quantity at successive known points of a cycle by suitably designed contact makers which can be arranged to close a measuring circuit at those points; these methods are quite useful, but involve the assumption that the successive cycles remain the same during the time of test. The most usual method is to use the instrument known as Duddell's Oscillograph the operation of which is as follows. If we provide a moving coil galvanometer in which the natural period of swing is very small compared with the period of the harmonics with which we have to deal in the curve we wish to investigate, the angular displacement of the coil will be an almost exact measure of the instantaneous value of the current it is taking. The galvanometer can be arranged to measure pressure or current at will in the ordinary manner by means of high series resistances or suitable shunts. In order to attain the desired high period it is necessary to have a minimum moment of inertia and a maximum controlling couple, this is attained as far as possible by making the coil of two tiny parallel strips as shown in Fig. 18, the strips being controlled by tension along them and being so arranged that the current flows up one and down the other, so that if the strips are in a magnetic field which is perpendicular to them they will move in opposite directions, and if a tiny mirror is suitably fixed to the middle of the two strips, it will tilt to and fro; a beam of light incident on the mirror will thus be expanded into a very narrow band; the excursions of the spot from its zero position are practically proportional to the strip-current. The diminution of the size of the coil and the high controlling couple exerted by the large tension applied to the pair of strips diminish the sensibility unless the magnetic field be very strong; this may be provided for by making the field magnet of the galvanometer a powerful electro-magnet. To obtain a record of the strip-current as a function of the time two methods are available; the simplest is to let a photographic plate fall quickly across the horizontally moving

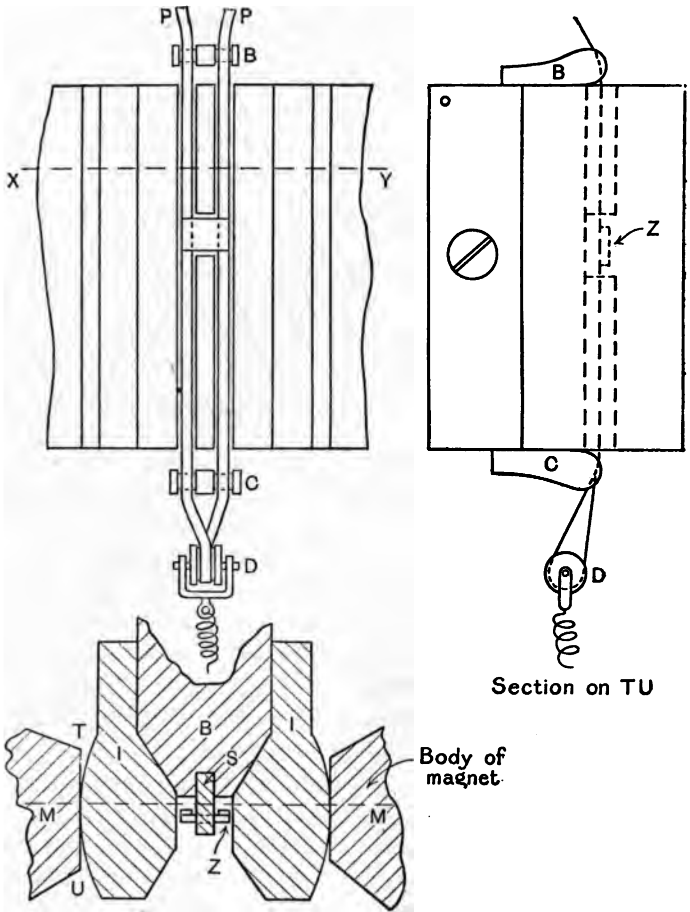

P
P
B
X
Y
C
D
T
B
I
I
S
M
M
Z
U
Section on XY
Body of magnet
B
Z
C
D
Section on TU

Fig. 18

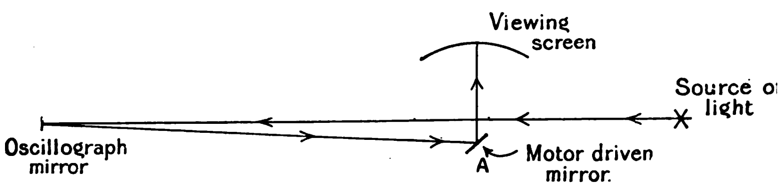

Viewing screen
Source of light
Oscillograph mirror
A
Motor driven mirror.

Fig. 19

beam of light, which thus leaves a trace on the plate, and for transient phenomena such as those produced on making and breaking circuits, this is the only method available. For continuous phenomena we can use visual observation which is secured as follows: in the return path of the beam is placed a pivoted mirror (Fig. 19) which turns the beam on to a screen where it can be viewed. If we can give a quick turn to the mirror, the beam will describe a curve on the screen. This turning must be so arranged as to be exactly proportional to the time and also to be such that no confusion between the observed curves can occur. These points are secured by reciprocating the mirror by a suitable cam which is itself driven by a specially designed motor of the type known as "synchronous," that is to say, one that exactly keeps step with the applied frequency of supply, from which source this motor is driven. To avoid confusion due to images made on the return stroke of the mirror, the motor carries a little screen so arranged as to cut off the incident beam during the return stroke.

12. Polyphase Armatures: Two Phase. Hitherto as a rule we have supposed that the dynamo armature carried only a single winding, the machine is then called a monophase alternator, but by far the greatest number of alternators are provided with two or three similar circuits on the armature arranged with a definite angular difference between their windings; we will now proceed to consider such "polyphase" machines.

Suppose we wind two armatures on the same armature core, so displaced relatively that the maximum E.M.F. occurs in the

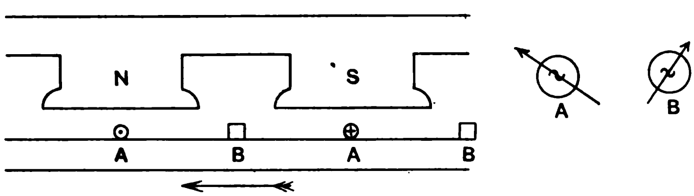


Fig. 20

second where zero E.M.F. occurs in the first, as is indicated in Fig. 20. We then have two equal E.M.F.'s produced between the two pairs of terminals differing only in that one has exactly 90° phase relative to the other. If we assume the field forms to be such as to give pure sine pressures, the E.M.F. of one being given by

$e_A = E_m \sin\theta$, that of the other will be $e_B = E_m \cos\theta$ as shown by the vectors. The two armatures being quite independent, we can link them in any way we choose, thus we can connect one end of

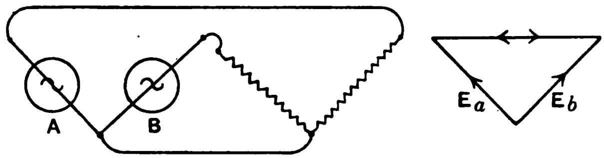


Fig. 21

each as shown in Fig. 21 and put them on loads: we will assume that the loads are identical or what is called "balanced," so that they will take the same currents at the same angles of lag. Hence the currents must be given by

$$i_a = I_m \sin(\theta - \lambda) \text{ and } i_b = I_m \cos(\theta - \lambda).$$

If the two armatures are quite independent, there is nothing to tell us what pressures exist between any two points one in either armature, since if the two systems are insulated from one another any difference of pressure may be present between them. But we have decided to link them as shown, it follows that between the free ends there is now a definite pressure, and in the common main the sum of the currents. The former can be found vectorially as in Fig. 21 since we have taken the pressures to be sinoidal, in fact it will be seen that the required pressure between the free ends must be the difference of the vectors for each E.M.F. separately, and hence is given by $\sqrt{2} . E$, where E is the *virtual* value of either, and has a phase angle of 45° to the component E.M.F.'s. Similarly the current in the common main must be $\sqrt{2} . I$, where I is either virtual or load current.

Such an arrangement is inherently asymmetrical since the three mains have pressure between them of the values E, E and $\sqrt{2}E$. But suppose we select the middle point of the load for the common point, we then get what is called a star connection, as shown in Fig. 22; this necessitates four mains, but the pressures between them are all equal. It will be seen that the virtual pressure due to each dynamo being still E, that between successive pairs of

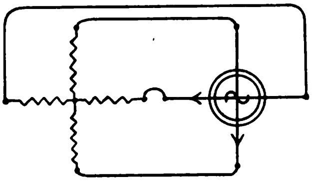


Fig. 22

mains, the "line" pressure, being the resultant of two pressures each $E/2$ in amount and at 90° phase will be $E/\sqrt{2}$. Here the currents are of course all equal, the phase of each load being the same.

But there is another method of producing this disposition: suppose we take four tappings at 90° apart on an ordinary direct current armature; between diametral sets we shall have the same result, namely equal pressures at 90°, but the actual condition of things is as shown in Fig. 23, since each quarter of the armature may be looked upon as a separate alternator. Such an assembling of the armatures is called a mesh connection, and it is really one of four phases. Indeed the previous star connection could have been equally well regarded as made up of four armatures half of each main one being regarded as a unit. With the mesh connection we must have the line E.M.F. between adjacent pairs equal to E, where E is the E.M.F. in an armature. When the connection (as supposed) is to a direct current armature having a D.C. E.M.F. of the value E_0 volts, this must also be the maximum value of the alternating E.M.F. between opposite mains, and hence the virtual pressure between them must be $E_0/\sqrt{2}$. But the figure shows that the latter pressure is $E\sqrt{2}$, and hence the relation between the line pressure and the D.C. pressure is $E = \frac{1}{2}E_0$. Of course these ratios are dependent on the presence of a sine field form; if the form be not nearly sinoidal, or if the harmonics in it are not well removed by the winding, the ratio will be different.

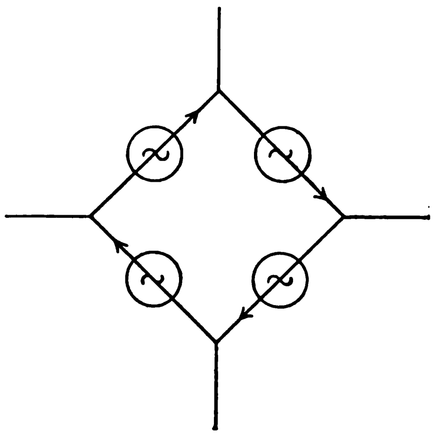


Fig. 23

The current I taken by the mains is the same for each; since to proceed from one mesh to the next we must pass through an armature, the current in the latter must be the vector difference between the two main currents, that is, with balanced load it must be $\sqrt{2} \cdot I$.

With a non-inductive load, not only is the line pressure at 45° to the dynamo pressures, but the current in the common line is at

45° to the dynamo current, and if the equal loads have the same lag angle, it will also be transferred to the armatures. In the ordinary monophase alternator carrying a current I at an angle λ to the pressure E the output at any moment is

$$E_m I_m \sin\theta \sin(\theta - \lambda),$$

and if E and I be the virtual pressure and current, it has a mean value $EI \cos\lambda$ with a double period fluctuation, passing from positive to negative and *vice versa* four times per period, so that the shaft couple also experiences similar periodic variations. The two-phase machine is quite different; there the instantaneous power if balanced is given by

$$E_m I_m \{\sin\theta \sin(\theta - \lambda) + \cos\theta \cos(\theta - \lambda)\},$$

and this reduces to $2EI \cos\lambda$, showing that the output, and consequently the shaft torque, is quite constant over the period, with no fluctuations at all.

If the curve or pressure of the armatures is not sinoidal, it will be seen that the shape of the line pressure curve and that of the armature may differ considerably when the connections are in star. The line pressure is the difference between the pressures in two adjacent armatures, and hence can be found by drawing the armature E.M.F. curve and subtracting from it a similar curve displaced by 90°. The way in which the harmonics appear can be found as follows: if any harmonic, say the qth, in one armature produces an E.M.F. given by $A_q \sin q\theta$, that in the other armature is

$$A_q \sin q(\theta + \pi/2),$$

and hence the difference, or line E.M.F., is

$$A_q \{\sin q\theta - \sin q(\theta + \pi/2)\}.$$

The bracket reduces to

$$2 \sin\frac{q\pi}{4} \cos\left(q\theta + \frac{q\pi}{4}\right), \text{ or } 2 \sin\frac{q\pi}{4} \sin\left\{q\theta + \frac{\pi}{2}\left(\frac{q}{2} + 1\right)\right\}.$$

The fundamental, for which $q =$ unity, gives an amplitude of $2A \sin\pi/4$ or $\sqrt{2}A$ as it must do; the others will have varying amplitudes and phases relative to the fundamental. It may be noted that every harmonic present in the armature reappears in the line pressure, for the condition for vanishing would be

$$\sin\frac{q\pi}{4} = 0 \quad\text{or}\quad \frac{q\pi}{4} = n\pi \quad\text{or}\quad q = 4n,$$

where n is any integer, and since only odd harmonics are present in the armature's E.M.F.'s, this cannot be fulfilled.

Apart from the larger specific output and its constancy, the great importance of the two-phase machine is that it enables gliding fields to be produced, as will be shown later on.

13. Three Phase. The true two-phase connection is, as we have seen, non-symmetrical, the symmetrical arrangements are really four phase. We will now consider a connection in which three phases only are employed, needing but three mains instead of four. To secure symmetry we must arrange so that the three

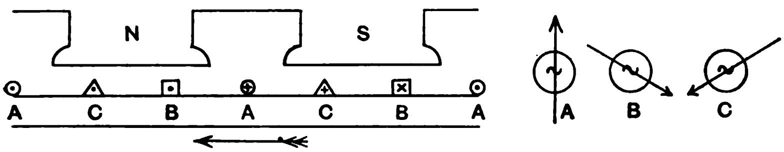


Fig. 24

armatures are displaced round the pole pitch at two-thirds of that pitch apart, as shown in Fig. 24 for a plain concentrated winding. This means that the three E.M.F.'s of the armatures with sine field form will be expressed by

$$e_a = E_m \sin \theta, \quad e_b = E_m \sin (\theta + \tfrac{2}{3}\pi), \quad e_c = E_m \sin (\theta + \tfrac{4}{3}\pi).$$

As before, the three armatures may be taken as independent entities and can be joined together in any way; there are two main methods of connection. We can join similar ends of each to a common point as shown in Fig. 25 when we have a star connection, the common point being called the neutral point; since the sum of the projections of the three E.M.F. vectors is always zero, the neutral point must remain at

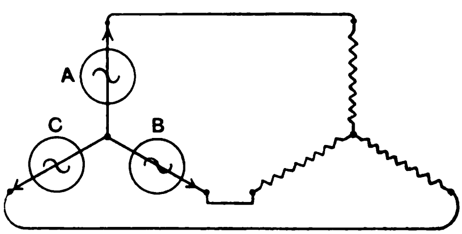


Fig. 25

zero pressure apart from external causes. The line pressure is found by proceeding from one main to the next viâ two armatures, and is hence the *difference* between their E.M.F.'s, or is $\sqrt{3}$ or 1·73 times either. If the loads be balanced, the same on each pair of lines, and if they be non-inductive with a current I, this current in the line is the same as that in the armature and is at 30° phase to the line pressure, and in phase with the armature

pressure; any angle of lag is common to all three circuits. In the star connection, then, if E and I refer to any armature, the line quantities are $\sqrt{3}E$ and I.

But we can also join the armatures end to end in the proper order, as shown in Fig. 26, forming the mesh or "delta" connection.

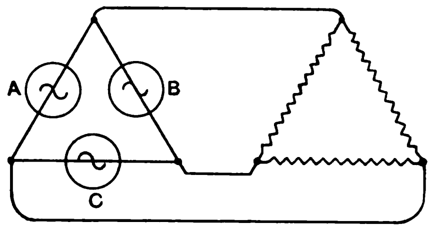


Fig. 26

Since with sines the instantaneous projection of the sum of the three vectors is zero, there is no nett E.M.F. acting round the closed circuit formed by the three armatures; further, the line pressure must be the same as the armature pressure, but the current in a line is the vector difference of two neighbouring armature currents which are at 120°, and hence is $\sqrt{3}$ times either, so that with the mesh connection, if E and I refer as before to an armature, the corresponding quantities for the line are E and $\sqrt{3}I$.

In both star and mesh the power is quite constant as in the two-phase machine, and has the value $3EI \cos \lambda$. Sometimes it is necessary to express the power in terms of the line pressures and currents: in the star the current is I and the line pressure E_L. Since the latter is $\sqrt{3}E$ we have

$$\text{Total power} = \sqrt{3}E_L I \cos \lambda.$$

Similarly with a mesh connection the expression in terms of the line current I_L is $\sqrt{3}EI_L \cos \lambda$.

Students sometimes find a difficulty in seeing that a definite state of line load, say non-inductive, is likewise carried by the armatures individually. It must be remembered that the suppression of the common main in the star, or the amalgamation of mains in the mesh, is so to speak an economical device. For example, the actual connection to the star load may be imagined to be as in Fig. 27, where the common main can be suppressed with simple sine currents, since no current

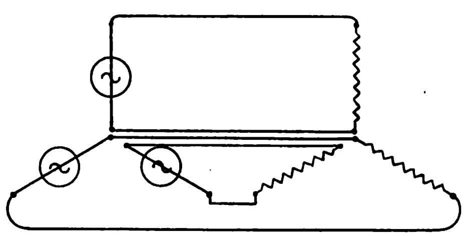


Fig. 27

would then flow down it. Similarly the mesh may be regarded as being like Fig. 28, the fusing of the pairs of mains being so to speak a matter of convenience. These figures show that the armatures are loaded up in accordance with the nature of the individual loads. Similarly with a mesh dynamo on a star load; as the latter is non-inductive, the current per phase is in phase with the pressure on the phase. But the main pressure, that due to the mesh dynamo, is at 30° to the phase pressure, and at the same time the currents coming down the (fused) mains combine into a single current, namely I, which has the same phase relation to the main currents. Hence the same result holds good.

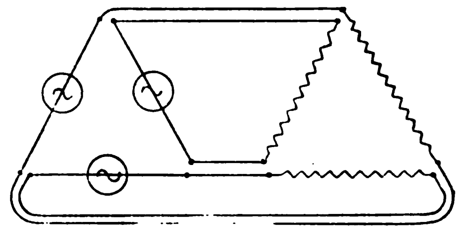


Fig. 28

With non-sine armature E.M.F. the line and the armature or phase E.M.F.'s will differ; the former is found by subtracting from one armature E.M.F. another similar E.M.F. displaced 120°; since the curve of E.M.F. is the same for its negative half as its positive, or repeats its shape with reversal each 180°, the result can also be found by adding two phase curves at 60°. We can readily find the effect on the harmonics, for if the qth in any armature be $A_q \sin q\theta$ the similar one in the next is given by $A_q \sin q\,(\theta + \frac{2}{3}\pi)$, hence the line pressure will be

$$A_q \{\sin q\theta - \sin q\,(\theta + \tfrac{2}{3}\pi)\},$$

which reduces to

$$2A_q \sin \frac{q\pi}{3} \sin \left\{q\theta + \left(\frac{q}{3} + \frac{1}{2}\right)\pi\right\}.$$

Thus the harmonics reappear with differing phase and amplitude; unlike the four-phase machine some are automatically wiped out. For, any one in which $\sin \frac{q\pi}{3} = 0$ cannot appear in the line pressure, so that all harmonics for which $\frac{q\pi}{3} = n\pi$ or $q = 3n$, where n is any integer, are absent from that pressure. It follows that *no* harmonic whose order is a multiple of 3 can appear in the pressure curve of a three-phase star connected machine with open neutral. This is a matter of much importance, as the third harmonic is sometimes rather large, and is often the cause of trouble. If in addition we

use on the armature one of the winding schemes which wipes out the fifth harmonic, we shall have a very fair approximation to a pure sinoidal E.M.F. between the lines. The non-appearance of the third harmonic will be evident from reference to Fig. 29 which shows the three fundamental sine E.M.F.'s of the three armatures together with the third harmonic; it will be seen that the latter is concurrent in all three armatures, hence if the difference between any pair is taken, when the main curves have a difference of phase of 120° the "thirds" in each are exactly anti-phased and hence cut out.

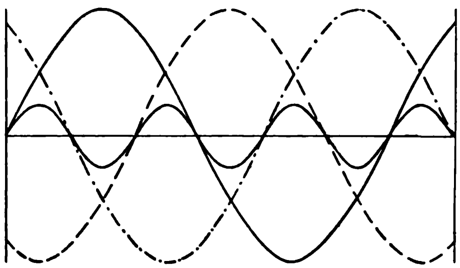


Fig. 29

The fact that the curves are non-sines carries with it the invalidation of a plane vector representation. Suppose the curve was given by $100 \cos \theta + 20 \cos 3\theta$, then from p. 20 the maximum value of the phase pressure is $(100^2 + 20^2)^{\frac{1}{2}}$ or 102. But only the fundamental is effective for the line pressure which therefore has a maximum value 173. We can give a geometrical representation of this: take a tetrahedron of height 20 and with its three basal edges of length 173, the sloping edges will give the phase E.M.F., the plane edges represent an E.M.F. of certain frequency, the height one of three times that frequency.

With the star connection all that the third harmonic does is to prevent the neutral or star point from remaining at a steady potential; it pulsates up and down in pressure owing to the co-phasedness of the third harmonic in each phase. When the neutral is open this does not produce any effect, but in some cases it is joined either to a similar point in the load or to the earth through an earthing resistance or reactance, and then there will be a current impelled down the neutral of three times the normal frequency. With a large harmonic this will cause trouble, and hence it is well to make it as small as possible in the armature.

When the connection of the armatures is in mesh, the third harmonics in each will be seen to be concurrent, so that they can conspire to circulate a current round the mesh itself, and this current may be a very large one.

14. Transformers. The use of transformers calls for little remark in respect to a sine E.M.F. but may be very briefly considered. All that has to be done is to combine the desired transformer pressure ratio with that necessitated by the different connections of the circuits. Thus, with mesh primary to mesh secondary, or star to star, the phase ratios are merely the ratio of the turns, but with star to mesh or *vice versa* the line to phase ratio comes in. Thus if the primary star be supplied at 3000 volts and the secondary mesh is to be 440 we must remember that the equivalent phase pressure of the primary is $1000\sqrt{3}$, and hence the transformation ratio of the primary and secondary turns must be $1000\sqrt{3}/440$: the matter is not worth labouring.

A further point is that we have two sets of possibilities, one as regards the linkage of the primary and secondary windings, the other as regards the connection of the secondary to its load. Thus with a star primary we may have the secondary mesh on mesh load or star on star load, but mesh on star load is not satisfactory with non-sine waves; similar conditions apply to the meshed primary. The secondary can be mesh on mesh load, or star on star, but should not be star on mesh.

As regards the magnetising currents some interesting points arise. Consider meshed primaries with a sinoidal primary line or phase pressure; the necessary magnetising current consists very largely of the third harmonic, and hence can only flow locally round the primary mesh. If we have a star wound primary connection with earthed neutral and star supply pressure, the necessary third harmonic currents can flow up and down the neutral wire. But suppose the secondary is also star and that the neutral is broken; the essential third harmonic current is now cut off, and the current in each primary must be practically a pure sine; but since the relation between flux and current is the same in shape as a cyclic curve, the flux cannot be a sine and consequently neither can the induced E.M.F. nor the phase pressure; in general this is rather peaked. If, however, the same transformer secondaries be in mesh, the necessary third harmonic has an open path, but in this case it is supplied by the secondary circuit instead of by the primary as in normal conditions.

In the above we assumed that the three transformers were quite separate and most systems employ this arrangement, especially when the units are very large, as it is simple to arrange for a spare transformer in case of breakdown. But it is possible to interlink

the magnetic circuits as well as the electric ones, so that there is a common magnetic circuit for the return fluxes. Like the E.M.F.'s the fluxes in the wound parts of the iron core will differ in phase by 120° and hence the yoke portions connecting the main limbs can have a smaller section than that of the separate transformers, giving a considerable saving in iron. But they are not usually made with truly symmetrical cores, being wound with a central limb and two side ones, from which it follows that the magnetic paths are not the same for every pair of coils, being longer for the two outer ones, and hence the three phases cannot take the same magnetising currents. A breakdown on a star wound transformer must throw it out of commission, though a mesh wound one will still act if the damaged phase has both its circuits disconnected and short-circuited, thus virtually supplying the system by means of an incomplete mesh of two sides only. The three-phase form of transformer saves a lot of space, and necessitates fewer high pressure terminals passing through the case, which itself is far smaller and less difficult to provide with cooling arrangements than three separate transformers, but the flexibility of the latter arrangement usually carries greater weight.

Certain points require care in such a three-phase transformer. For example, suppose it to be wound with star primaries, and that the neutral is open; the third harmonic current cannot flow, but if that neutral be grounded, it is free to flow. With three similarly disposed limbs on the transformer it will follow that the three third harmonic currents now active will be always circulating round the limbs in the same direction at a given moment, and hence will tend to force the appropriate flux through a magnetic circuit from top to bottom of those limbs; that is through a circuit which is practically air-cored; the current required will thus be very large. Hence the no-load triple frequency current taken may be excessive if the neutral be not open.

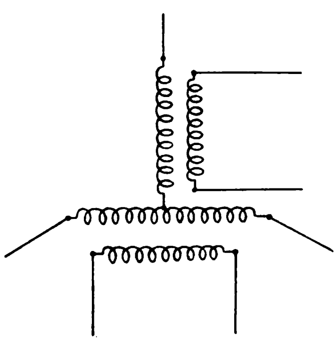


Fig. 30.

It is sometimes required to link up a two-phase system with a three-phase one and this can be done as shown in Fig. 30, known as Scott's arrangement. Two transformers

are provided, and the primary of one is attached to the centre of the primary of the other, the three free terminals going to the three phase lines. The secondary circuits will then be seen to give pressures at right angles or to form a two or four-phase circuit; this can be made to have equal phase pressures if the ratios of the transformers are properly adjusted. Thus, let the three-phase supply be at 3000 volts, the secondary two-phase at 440. It will be seen that one transformer must have the ratio 3000/440 while the other must have $\sqrt{3}/2$ times that ratio or $1500\sqrt{3}/440$. There is necessarily some slight asymmetry about this arrangement but not enough to be of serious moment.

15. The Armature's Resistance and Reactance. We must now proceed to deal with the second essential part of an alternator, namely its armature; this question falls into two parts, that concerning each armature as a separate entity, and the combined action of all the armatures. The two questions are to some extent linked together, but for the sake of getting clear ideas on the matter it is best at first to keep them quite separate. Hence we will now consider the two properties of any single armature which are, as it were, self-contained, and are but little dependent on the absence or presence of a field magnet, or on its form when present. These properties are its resistance and its reactance. The former as far as its normal or "direct current" value is concerned is easily found either by calculation from the known dimensions of the winding with a suitable allowance for temperature rise, or by direct test in the usual way. But with alternating currents there are parasitic eddy currents present in the wires and other places which waste power, and as a fair approximation they can be debited to the armature, which must therefore be credited with a supposititious resistance somewhat in excess of the value found by direct current tests. This increase is purely a matter of experience, but is something between 25 and 50 per cent.; the uncertainty in the value of the resistance, R, is in most problems of little moment, as the resistance is always small compared with the other armature constants in a modern alternator.

The second factor is the true reactance of the armature due to its leakage fluxes; these occur in various places. Imagine the armature removed from the bearings and thus free from the influence of the field. When a current flows in the wires in a slot as shown in

Fig. 31 fluxes will be produced in various paths round th which are collectively called the slot leakages. Such flux (1) the head leakage, side leakage above the and (3) the leakage fro to side of the slot whe wires are situated. Le (1) and (2) are impel the full magnetomotiv of the slot winding, th netomotive force for 1 (3) will diminish towa

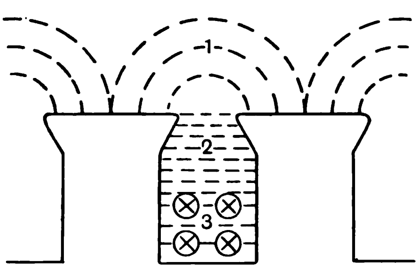


Fig. 31

root; these stray fluxes can be calculated approximat empirical formulae to be found in books on design. The le (2) and (3) across the slot will be but little affected wh armature is in place, but the head leakage (1) may be so af so that if we measure in any way the reactance of the ar per phase when in place, it will be found to depend to some on its position relative to the field magnet. Another place stray flux can be produced is round the end connecting w which the current is acting on a purely air-cored circuit: th can also be found roughly by approximate formulae, but determined for any special type of winding by direct measu of the reactance of sample coils having the same end conn but different lengths of slot. By such means an estimate reactance of any armature can be made. In general it ar to such a value as to absorb from 5 to 10 per cent. of the te pressure at full load current.

The determination of the true reactance by direct test from simple. It will be seen that the principal parts of t leakage and the flux surrounding the connecting wires will little altered if we replace the armature on the polar rin leakage from top to top of the teeth will be somewhat af since under the poles it can find a slightly shorter path from to tooth by passing up into the polar face and back again i of curving across from tooth to tooth as must be the cas the pole is absent. But the head leakage is but a small part total, and hence we are justified in assuming that there i alteration produced in the true reactance by isolating the ture. Under this supposition the reactance can be measur

any other by the use of ammeter, voltmeter and wattmeter and an auxiliary supply at the proper frequency; in fact the measurement of pressure and current will usually be sufficient since the reactance is generally many times the resistance. The true reactance of any individual armature will always be denoted by X_a.

As far as concerns the resistance and reactance an armature can be treated like any other ordinary air-cored reactance coil, namely the current will demand a pressure that is in phase for the resistance and one leading 90° for the reactance.

16. The Armature as Magnet. When, however, we come to deal with the other effects of the combined armatures on the field, effects which are purely magnetic, we are met with some new considerations owing to the fact that the windings on an armature are dispersed instead of simply surrounding the active iron as with transformers or with the field magnets of the alternator in its commoner form; this is due to the fact that in addition to a time distribution of flux we have to take into account a space distribution. Consider first the very simple case shown in Fig. 32 where a uniform air-gap exists between two concentric cylinders, and let a single coil of wire of N turns with a steady current I be placed as shown: a flux of magnetism will result having the general direction of the arrows. We can practically take it that the reluctance is mainly due to the air-gap, and hence, as a close approximation, that all the magnetomotive force (M.M.F.) due to the coil is expended on the gap.

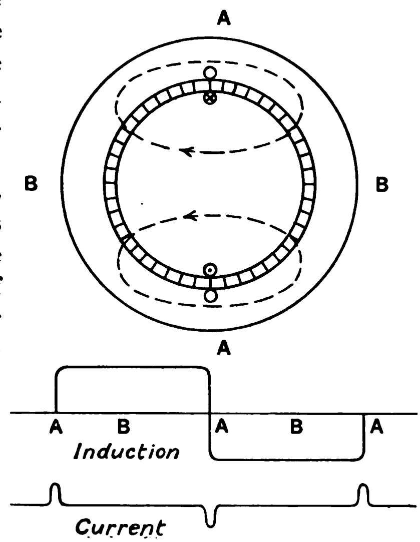


Fig. 32

A simple inspection will show that this M.M.F. is the same for all paths threading the coil, and hence that the M.M.F. in the gap must be uniform all round except at the small fractions of the

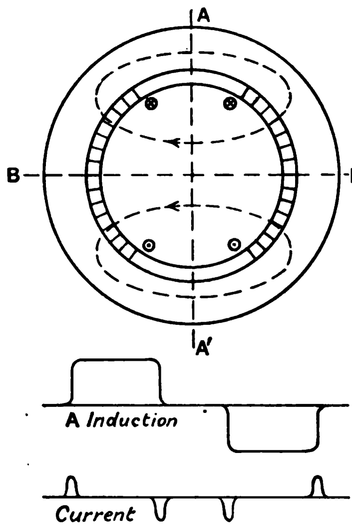


Fig. 33

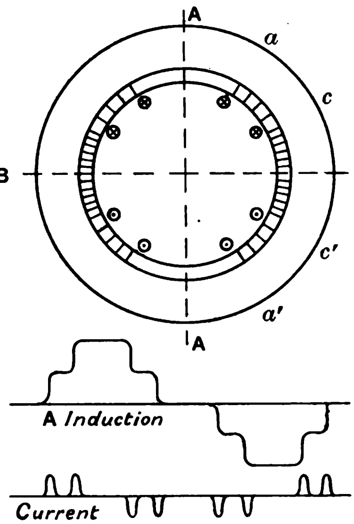


Fig. 34

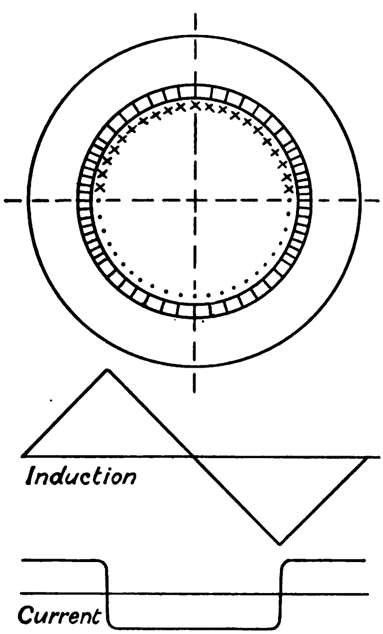


Fig. 35

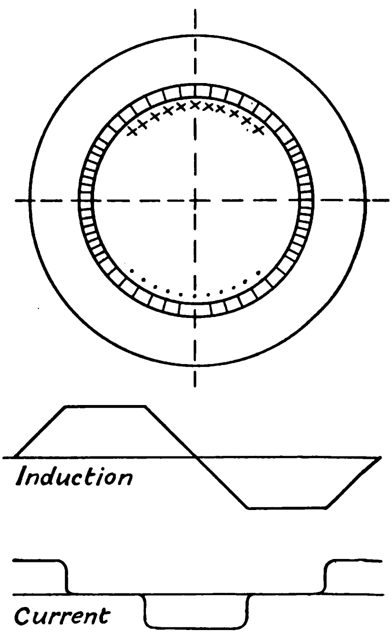


Fig. 36

circumference taken up by the coil. We can therefore draw a curve, Fig. 32, showing how the M.M.F. varies with the distance from any selected point say A. For brevity this curve of gap magnetomotive force will be called the G.M. curve. Further we can correlate with the G.M. curve another curve showing how the current-turns are spaced round the circumference. It should be noticed that it is indifferent whether the windings be as shown, on the inner cylinder, or, as shown dotted, on the outer cylinder, the effect is very nearly the same.

We can arrange the same total turns carrying the same current in many ways, some of which are shown in Figs. 33 to 36, where dispersed and distributed windings are shown each with its appropriate G.M. curve and current curve; if each arrangement fulfils the condition of having the same total current-turns, it follows that whatever be the shape of the G.M. curve they will all have the same maximum value; this is an important point to bear in mind. Strictly speaking the distributed windings should have the G.M. curve made up of tiny steps but these are smoothed out in the figures. It will be seen that the current curves coincide with the slopes of the G.M. curve, and that this is a general relation between the two can be shown as follows.

Consider two adjacent points round the air-gap distant dx apart, and let the magnetic forces at those points be H and $H + dH$. We know from first principles that the line integral of the magnetic force round any circuit is $4\pi \times$ (current-turns inside) when absolute units are used. Also if m be the turns per cm. those in dx are mdx, and as the current in a wire is I, the current-turns inside the points are $mIdx$. But if a be the width of the gap, the integral of the magnetic force round the dotted circuit is seen to be adH, since all the magnetic force is used up in the air, so that we have $adH = 4\pi mIdx$. But if we denote by M the M.M.F. acting at

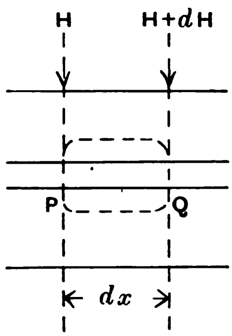


Fig. 37

any point round the armature, since no magnetic force is expended on the iron, it must follow that adH is the small increment of M.M.F. due to the element dx, or that $dM = 4\pi mIdx$. Hence if we measure the M.M.F. round the armature, starting from a point at which the magnetic force is zero, the M.M.F. at any point at a distance

x along the gap is given by $4\pi m \int_0^x I dx$. This is the equation to the G.M. curve and shows that it is the integral of the current curve. With any practical form of winding, the latter is symmetrical about its mid-point, and it follows that the G.M. curve will have its maximum when we have integrated half way round the current curve, or when we have included half the wires concerned or all the turns; denoting this number of turns by N, we see that the maximum of the G.M. curve must always be $4\pi IN$ which is quite a fundamental point.

It is very usual to measure the M.M.F.'s simply in ampere-turns instead of in $4\pi/10$ times that number, as all practical questions involve that reckoning, so that we shall very frequently use this method of measuring the M.M.F. without any reference to the omission of the $4\pi/10$. No confusion will arise if a little attention is paid to the matter.

If this band of G.M. is allowed to be operative and is not counterpoised by external means, it will produce at every point an appropriate induction. If the gap be uniform as hitherto assumed this induction is readily found; for consider a square cm. on the surface of the armature, the reluctance of the little bit of magnetic circuit on which the M.M.F. is acting will be simply the radial distance between the two cylinders, or will be a, and the flux passing is the induction B; hence the ordinary relation between flux, reluctance and M.M.F. leads to $B \times a = M$. But we have $M = aH$, where H is the magnetic force anywhere, so that $B = H$ as is necessarily the case in air. It follows that we can find the induction produced by dividing the M.M.F. expressed in absolute units by the air-gap depth in cm.; if the M.M.F. be in ampere-turns we must also multiply by $4\pi/10$, that is $5/4$, nearly.

Even with non-cylindrical gaps a similar relation holds, but we then have to guess at the form of the reluctance circuits and allow for the variation of magnetic force along them as their cross sections vary, so that the value of a is not sufficient, but the G.M. curve is still applicable to such arrangements.

17. The Armature's Magnetic Field. We know that the most easily dealt with curve of G.M. would be a sinoidal one, such as is shown in Fig. 38, but this requires that the current density, or what is the same thing in a practical winding, the turns of wire

er cm. round the armature, should be distributed according to le ordinates of a cosine curve. This might possibly be arranged if le whole apparatus were not required to rotate, but is quite npossible in any practical armature. It follows that we cannot ctually obtain a sine G.M. curve in real machines, but it can be lown that such a curve does really exist to a high degree of pproximation in real machines under working conditions, so that e will for the present take this for granted, and assume in our rther investigation that the winding is accompanied by a G.M. irve which is sinoidal round the armature, and we will now proceed apply this assumption to the case of armatures which are rotating

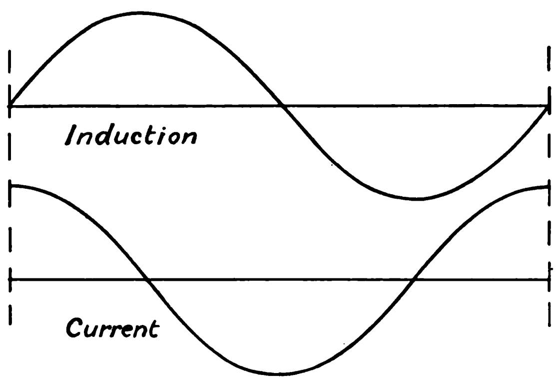


Fig. 38

d are thus supplying their own currents instead of having the rrent supplied from outside as we have hitherto considered. irther, the currents in the armature will be alternating, and have periodicity the same as the R.P.S., since we will still for simplicity ke the two-pole form for purposes of illustration.

The first thing is to settle on the relative position of the armare and field magnet at the instant when the current in an armare has its maximum value, and this must depend on the nature the circuit the armature is supplying. Let us at first take a onophase armature with a simple winding as indicated in Fig. 39. irther, let the E.M.F. produced by the polar flux, the so-called ominal E.M.F., have its maximum when the wire is exactly at its id-pole position; then if we suppose that the whole circuit supied by the armature is entirely devoid of anything but resistance, iat is to say, not only is the outside circuit a pure resistance, but

the armature itself is practically devoid of reactance, current must attain its max simultaneously with the E.M also when the wires are at mid as shown in Fig. 39. Such a c tion is expressed by sayin E.M.F. and current are in phas if the angle of phase diffe between the E.M.F. and the cu is denoted by ψ the value of angle is then zero: this phase must be carefully distinguished the phase angle of the load and it is to the angle ψ th questions of current-phase w referred. Such a condition being zero cannot be produc practice owing to the imposs of depriving an armature c true reactance, but must be as an ideal limit of the pro We will denote the G.M. cur an oval surrounding the arm such that the intercepts be the circle representing the arm and the oval, when taken ra give the appropriate values c G.M. at the several points outside oval is drawn with lined boundary for that part the G.M. tends to produce a polar flux and a dotted bou where it tends to produce a polar one: the oval must be posed to rotate with the arm and as it rotates its maximum or its mid-ordinate will vary the current flowing. If M c the maximum at the moment in the top figure, when it has turned through the angle θ

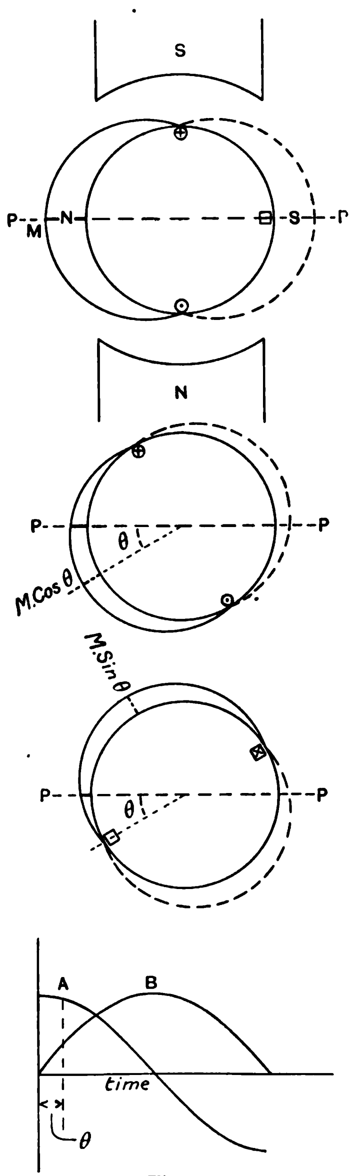


Fig. 39

maximum will be smaller and have become $M \cos \theta$ as shown in the second figure.

If the winding be fully concentrated and of full pitch, there are two complete semicircular arcs round the armature such that at any point in the upper half of the armature a wire must at any time have a downward flowing current as shown by the cross or arrow tail on the single representative wire, while a wire in the lower half will similarly everywhere and at all times have an arrow head. It follows from this that whatever be the phase distribution of the individual wires round the two arcs, the nett magnetic effect is one along the axis *PP*. Further each wire in one of the arcs will have a similarly situated return wire in the other arc, and everywhere and always the two will consequently give a magnetic effect acting along *PP*; this is quite fundamental and must be made familiar before going further. Indeed it will not signify in the least how the wires are made up, whether into one, two or three phases, they must in all conditions give a resultant band of M.M.F. which of necessity preserves its direction always along the line *PP* which is itself at right angles to the position of any coil at the moment of maximum current. It follows also that no reversal of the direction of the M.M.F. can occur; it may pulsate or vary in any sort of way; in certain conditions as we shall shortly see it may be quite constant and immovable; but it must always have a fixed axis however its maximum alters, in fact it may pulsate but can never alternate. If the winding be dispersed, but still full pitch, the coil when carrying its maximum current will occupy a position such that its centre is at the position shown by the arrow tail in the top figure. Similarly, when it has zero current it must, from symmetry, occupy a similar position relative to the line *PP*. It follows that the portion of the semicircle which can be occupied by the arrow tails and heads is reduced from its full value of half the circumference to one occupying an angle of $(\pi - 2\eta)$, where 2η is the angular breadth of the winding. But within that somewhat restricted arc, all the above will still apply. It will still be true for short chord windings if suitable alterations are made in the specification of the arc.

18. The Two-Phase Machine. Now let us return to Fig. 39 and let the coil move through the angle θ. We have seen that the maximum of the G.M. band will be reduced to $M \cos \theta$, but the

figure also shows that the intercept along the line PP where that line cuts the G.M. oval is such that it is smaller than the maximum of the band at that moment by the same factor namely $\cos\theta$. It follows that when the armature has turned through the angle θ the value of the G.M. acting along the fixed axis PP is $M\cos^2\theta$.

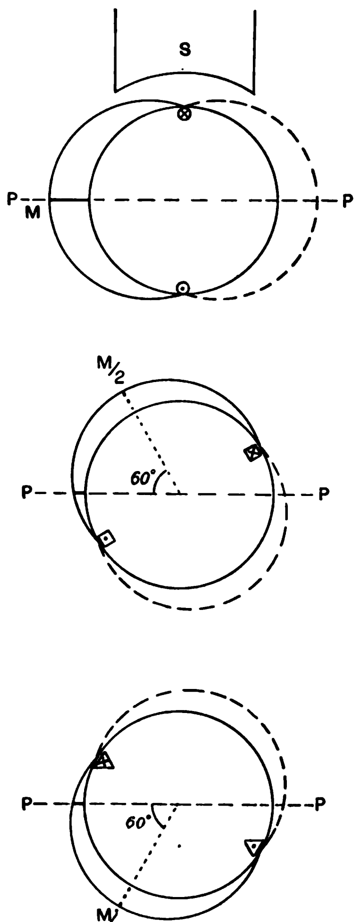


Fig. 40

As this can be written in the form $M(1 + \cos 2\theta)/2$, it also shows that the effect consists of two parts, a constant one $M/2$ and one that varies at twice the periodicity of the armature E.M.F.; this matter will be elucidated later on, but we will first see if things cannot be simplified by what is at first sight a complication, namely by supposing the machine to be a two-phase one with the second armature displaced exactly 90° round the core as shown by the square wires, and supplied therefore with a current which is also exactly at 90° phase to the first, so that if the first armature, which we will call A, has a current (as we have supposed) which varies as $\cos\theta$, that in the other armature, which we will call B, will vary as $\sin\theta$.

As regards A, from what has gone before we see that it will contribute at P a G.M. which is given by $M\cos^2\theta$, where M is of course the turns in the armature A multipled by the maximum current in it, namely I_m. But the other armature has now come into operation as shown below the first; the belt has begun to grow, and its maximum, directed as shown, will have attained the value $M\sin\theta$. But an inspection of Fig. 39 shows that the part of this available at P is the maximum multiplied by $\sin\theta$ and hence its contribution is $M\sin^2\theta$. Thus

he two together will provide at P a M.M.F. given by

$$M(\sin^2\theta + \cos^2\theta)$$

r M exactly as before. It follows that the belt remains quite fixed ı space while the armature rotates, and alters neither in size, ›rm nor position. Thus with two phases on the armature the :fect is perfectly simple; they conspire to produce a fixed and efinite belt of M.M.F. which has a maximum value exactly the ıme as that due to *either* belt alone at its moment of maximum ırrent, namely $M = NI_m$, where N is the turns on one armature.

19. **The Three-Phase Machine.** We shall now see that a milar effect is produced with three phases. First consider the ıstant when the current in one of the phases A is a maximum ith the same condition of circuit as before, namely when the ırrent is absolutely in phase with the pressure due to the magnet's ux. The condition as regards that phase is then as shown in the pper figure of Fig. 40. At that instant the other phases will have he positions shown below, both B and C carrying their appropriate ırves of M.M.F.; but at the instant that A has its maximum curent, B and C will only have half that current, hence the maximum f their curves is but half that of A. Further, while the contriution of A is its full value for the position of the armatures amely $M = NI_m$, that of the other two as shown by the correponding intercepts bears to the maximum of each the ratio cos 60° r $\frac{1}{2}$, so that the contribution of each towards the magnetic force t P is $M/4$ and hence the final value for the M.M.F. at P due to ll the three armatures has a maximum value of $3/2M$ instead of M only as in the two-phase armature. Now suppose the top rmature moves through the angle θ as do the others, from what e saw with the two-phase machine and from what we have just ›und, it will follow that the joint contribution towards the ıagnetic force at P is given by

$$M\{\cos^2\theta + \cos^2(\theta + 120°) + \cos^2(\theta + 240°)\},$$

hich reduces again to $3/2M$, showing as before that the belt f magnetic force remains constant in size, shape and position.

This question will be considered in more detail later on for those udents who desire a fuller investigation, and the matter will then e proved in a more general way. Meanwhile we may take it as emonstrated that in all polyphase machines the total magnetic ffect compounded of the separate ones of all the armatures when

the machine has a balanced load results in a band of M.M.F. practically identical in its nature to that existent in an ordinary direct current dynamo. Indeed the result is an immediate consequence of the facts proved on p. 30, namely that the electrical output of a polyphase machine is perfectly constant, for if the output be constant so must be the input, and since the machine is rotating at constant speed, the torque demanded must be constant. But such a torque can only be brought into existence by the mechanical force between the magnetic field due to the poles and that due to the armature, and since the former is fixed and constant, so must the latter be; so that the magnetic effect of the polyphase machine's armatures is necessarily the same as that of a direct current machine.

20. The Monophase Machine. We will now return to the monophase machine and see if it can be brought into line. It was indicated that the pulsating effect which we saw must exist was one of double frequency superposed on a steady one, each of half the maximum value of M. Suppose then that the top figure of Fig. 41 is a representation of the fixed band of half maximum value, and that we imagine another band of the same *constant* maximum, namely $M/2$, to be rotating round on the surface of the armature at the same speed that the armature itself is rotating in space, so that the band rotates in space at *twice* the armature's speed. At the instant of maximum the two coincide and produce the normal maximum band of Fig. 39, but after one quarter of a period, that is when the armature has turned through a right angle, there is no current and hence there should be no M.M.F. But since the rotating belt travels twice as fast as the armature, it will have made half a complete turn, and hence will be in a position to annul exactly the fixed band at that moment; if we take any intermediate

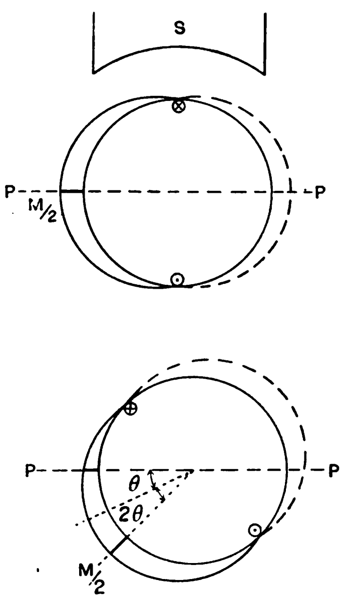


Fig. 41

position as shown in the lower figure of Fig. 41, corresponding to an armature rotation through any angle θ, the belt will have turned through 2θ, so that while the fixed band will contribute the amount $1/2M$ to the M.M.F. at P, the rotating band will contribute the amount $M \cos 2\theta/2$; the sum is therefore $M(1 + \cos 2\theta)/2$ or $M \cos^2 \theta$ as was shown at first. Hence the monophase machine can likewise be looked on as producing a steady band of M.M.F. but one of half the maximum size, with a superposed band rotating twice as fast as the armature. The rotating band will then have the power of inducing currents in the whole of the field magnet circuit, and in particular in the winding on the magnets, and such a double period induced field current is shown in monophase machines (see Fig. 82), though it is quite absent from polyphase ones. The double period variation introduces a special set of difficulties in the ordinary alternator which are entirely absent from the polyphase form.

It may help in getting a clear notion of the action of a polyphase dynamo to reverse the argument as follows: we see that a monophase armature produces a stationary field of maximum value $M/2$, and one rotating twice as fast but with the same maximum. Suppose we have p symmetrical phases on the armature, each carrying the same current in magnitude and phase. There will then be p superposed stationary M.M.F. curves giving a resulting one whose maximum is $pM/2$, together with p rotating ones of the same maximum but of double period. If the latter are suitably related in space, as with the 2, 4, 6 etc. phase conditions, the several components will just neutralize one another, leaving the $(pM/2)$ belt at rest as the only one physically present.

21. The Effect of the Stationary Magnetic Belt. The position of the resulting stationary belt of G.M. produced by the armatures relative to that due to the field magnet winding is settled by the phase angle between the E.M.F. due to the polar flux (or the nominal E.M.F.) and the armature current. As we have seen the two belts are exactly in quadrature in space when the current in each armature is in phase with its E.M.F.

If the phase of the current alters by an amount ψ relative to the mid-pole, its belt will shift round by the same angle. Suppose then that the current is flowing through a circuit in which that phase angle is a lag of 90°, that is, in a very highly inductive circuit:

the belt will shift round in the direction of rotation as shown in Fig. 42 in order that it may accommodate itself to the fact that the current does not attain its maximum in the several armatures until one-fourth of a revolution has elapsed. The figure shows that in this position the belt of M.M.F. directly opposes the winding on the field and produces a fully demagnetising effect instead of one across or at right angles to the main flux. Again, if we could load the armatures on condensers, or in some equivalent way so that a leading current of exactly 90° advance in phase over the mid-pole position occurred, the belt would shift round to the opposite position to that of Fig. 42 and thus help the winding on the poles.

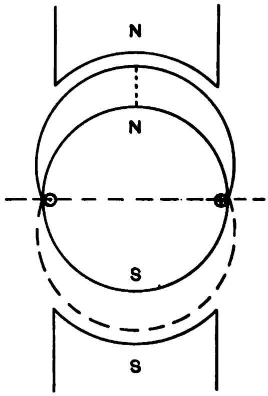


Fig. 42

It follows that a fully lagging current in each armature produces a demagnetising effect, a fully leading one a magnetising effect, and a current on a non-inductive load or with zero value of ψ a cross magnetic effect; these points are also quite fundamental. For any other phase difference the belt will take up an intermediate position but always such that with an angle of phase ψ between the nominal E.M.F. and the currents the angle between the belt's axis and that of the poles is $(90° - \psi)$. The effectiveness of this stationary band of M.M.F. will depend on the nature of the magnetic circuit on which it acts, and this again on the form of the field magnet surrounding it. If that be of the ordinary type with salient or discontinuous polar projections, the magnetic circuit acted on when the band is collinear with the polar axis, that is when it is magnetising or demagnetising, will be the same as that acted on by the polar winding itself, but when the band is in the position of non-inductive load, it will be acting on a totally different circuit namely that between the poles which has far more reluctance. For a fairly complete investigation of this effect it will be necessary to consider the matter in some detail as is done later on, but a very great deal that is of considerable practical importance can be established by supposing that the magnetic circuit surrounding the armatures is completely symmetrical somewhat like the figures on p. 40, or that the magnet, like the armature, has no salient poles,

but is of cylindrical form inside or surrounding the armature as the case may be. Indeed with turbo-alternators this is nearly true; the "rotor" or field magnet is very often made in such a cylindrical form, and the resemblance to an armature is accentuated in many machines by the fact that the magnet is wound in a distributed manner very like the winding on an armature. Hence we will assume for the present that the air-gap is quite symmetrical all round, and that the armature and field magnet form concentric cylinders. If this be so, the magnetic effect produced by the armature must be independent of the position of its band of G.M. relative to the field magnet, or in other words independent of the angle ψ.

The effect produced by the stationary band may be looked on in two independent ways, and we will now proceed to consider the first. We have seen that a two-phase machine produces a sinoidal band of current-turns which has a steady maximum value $I_m N$, where I_m is the maximum current in any armature and N is the number of turns in that particular armature.

It is much more convenient to express the effect in terms of the virtual current I in each armature, so that $I = I_m/\sqrt{2}$, and the total number of peripheral wires on the armature counted irrespective of the method of connection; if we denote that number by W, since the number of peripheral wires is four times the number of turns per phase with a two-phase machine, the maximum ampere-turns on the belt are expressed by $\frac{\sqrt{2}}{4} WI$ or $0{\cdot}35WI$.

A precisely similar expression applies to the three-phase dynamo, for we saw that the maximum ampere-turns were then given by $3/2NI_m$, and using again I for the virtual current per armature and W for the total peripheral wires in all the windings, since we have $W = 6N$ we once more obtain the expression $\frac{1}{2\sqrt{2}} WI$. Even the monophase machine falls into line, for we have seen that the steady effect is due to half the maximum current so that if we still reckon the turns in terms of the total peripheral wires, and the current in terms of its virtual value, we once again arrive at the expression $\frac{1}{2\sqrt{2}} WI$ for the armature ampere-turns; thus with the sine G.M. curve it is quite immaterial which form of dynamo we take, provided the specification of turns and current be as above; the formula $0{\cdot}35WI$ applies to any one.

The above of course only applies to the special form of G.M. curve assumed to exist, namely the sinoidal: the consideration of other forms will be taken later on, but the following simple way of determining the absolute maximum armature turns carried by a three-phase machine may be given as an illustration.

Suppose we consider a three-phase armature at the moment when the current in one phase is a maximum; that in the others must then be one-half that value. If the virtual current per phase be I and the peripheral wires number W, since one-third the wires are carrying full current and the other two-thirds are carrying one-half the current, the ampere-turns on the armature must then be

$$\sqrt{2}\left(I\frac{W}{2}\frac{1}{3}+\frac{I}{2}\frac{W}{2}\frac{2}{3}\right) \text{ or } 0{\cdot}47\,WI.$$

Now consider the moment when the current is zero in one phase, it will have 0·866 of its full value in the other two, hence the ampere-turns carried by the armature will be

$$\sqrt{2}\left(0{\cdot}866I\frac{W}{2}\frac{2}{3}\right) \text{ or } 0{\cdot}41\,WI.$$

These values are at times differing by only one-twelfth the period and it will be a fair approximation to assume that the mean or $0{\cdot}44\sqrt{WI}$ gives the constant maximum value of the armature's maximum magnetic ampere-turns without consideration of the effects of distribution.

It may be noted that some machines possess both a cylindrical air-gap and a field magnet wound in a distributed manner, that is to say, with a slot winding like the armature, this is frequently done in turbo-alternators; then both the field form of the magnet and that of the armature will be very approximately sinoidal in distribution forming bands of M.M.F. which, with non-inductive load, are at right angles in space. With this form of pole, the resulting band will also be sinoidal and the armature effect is then practically equivalent to sliding the resulting sinoidal band round the air-gap. This results in the extreme degree of simplification that is possible, and the results obtained are in fair agreement with the theory that will follow. With such distribution of M.M.F.'s it is permissible to deal with their combination by vectorial means.

Thus we can always deal with the effect of the armature by supposing that in every dynamo it produces a fixed belt of M.M.F. which has its axis at right angles in space to the position of maxi-

um armature current; when we wish to take into considera-ɔn the possibility of distributions of M.M.F. differing some-hat from the assumed one, and in particular when the ·erage effect of the band is required, we can replace the definite ımerical constant 0·35 by a general one and for this purpose ɜ will use the symbol ∇, so that the armature ampere-turns are ven by ∇WI. One method of dealing with the armature's de-agnetising action would be to annul it by means of an opposing M.F. supplied by the field magnet circuit. If the turns on all the ld coils number T and if we supply an *extra* amount of current er and above that required by the dynamo to give its desired M.F., and denote that extra field current by J_D (the letter J will kept for field currents), the value of that current must be given $J_DT = \nabla WI$. This is a very useful and important way of dealing th the question, and will be taken up again later on; meanwhile ɜ will proceed to develop the second method of dealing with the mature's total magnetic action, as it gives a concise and useful presentation which is both sufficiently simple to avoid the ne-ssity of complicated diagrams and is also sufficiently accurate practical use in most circumstances.

22. Synchronous Reactance. We have seen that the arma-res conspire to produce a fixed and definite band of M.M.F.; let further suppose that this band is actually operative in producing x so that it will produce an induction in the cylindrical air-gap its own account just the same in kind as that due to the field ıgnet. This induction is equal to the ordinate of the current-turn rve interpreted as M.M.F., divided by the air-gap depth if we sume that the principal reluctance encountered by the armature's M.F. is that of air-gap only. Thus, coincident in space with the nd of M.M.F. there will be another band of flux in addition to at due to the field magnet, and likewise stationary in space. is band is cut by each of the rotating armatures exactly as is ɜ field magnet's flux, and hence another E.M.F. is produced ɜreby in each armature. This E.M.F. is everywhere proportional the new flux, which is itself practically proportional to the rrent in any armature; and its position is that of the flux nd, namely exactly in quadrature with the maximum current. other words, this new E.M.F. is in effect exactly equivalent to an tra reactance in the armature, and this will be denoted by X_A.

From this aspect of the problem we replace the magnetic reactions of the armatures by a physical reactance in each, and on adding this new reactance to the actual true reactance X_a of each armature that it possesses in virtue of its own local leakage fluxes, we finally arrive at considering the whole of any one armature's magnetic action to be covered by supposing it to possess a definite total reactance X such that $X = X_a + X_A$. This is called the Synchronous Reactance of the armature, and together with the resistance gives the two fundamental constants by which the armature's action is at present to be explained and illustrated.

The concealment of the direct action of the armature in the same constant as that including its true reactance is likely to lead to misconception unless care is exercised. It must always be remembered that the use of the synchronous reactance in any problem implicitly carries with it the assumption that the effect of all the armatures on the field magnet is being taken into account; it is often supposed that this is then being left out of consideration which is not the case at all, in fact it is probably being somewhat overestimated when the Synchronous Reactance method is used.

The resistance and synchronous reactance can be combined into a single constant, the Synchronous Impedance, given by

$$Z = (R^2 + X^2)^{\frac{1}{2}},$$

which together with the internal phase angle given by $\tan \alpha = X/R$ specifies the two constants in a manner which is often more convenient than the direct use of R and X.

The fact that the apparent reactance of the armature is a complex accounts to some extent for the difficulty met with when we try to determine the true reactance of the armature *in situ.* If we attempt to conduct this measurement the result will depend on the actual X_a and some particular armature magnetising effect correlated with some special value of X_A. In Fig. 43 is given a curve showing how the apparent reactance of an armature, measured in the ordinary way by volts and amperes, varies with its position relative to the field, the armature having two slots per pole. The result differs also with the condition of the field, since the reluctance of the machine depends on its excitation; the apparent reactance is considerably larger with no excitation than with full owing to the diminished permeability in the latter condition. The direct effect can be somewhat diminished by conducting

the test with the fields short-circuited, thus to some extent preventing the entry of the flux into the field circuit. By rotating the armature during the test by means of an auxiliary motor, the various orientations are averaged out and a more satisfactory result obtained, but in any case such a test cannot give more than a rough approximation to the reactance of the armature.

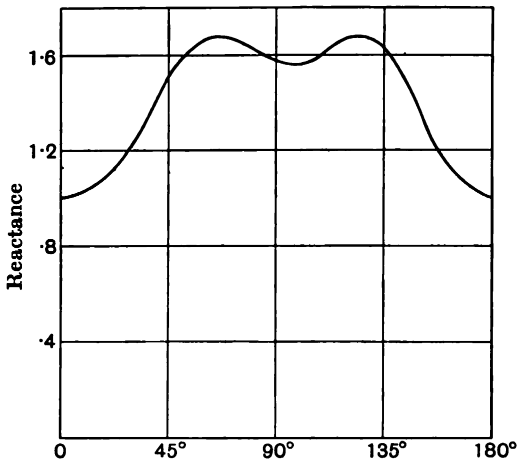


Fig. 43

23. Saturation Curve. We must now consider a method by which the synchronous reactance can be found; it involves the determination of two characteristic curves of the machine. The first is the ordinary Saturation Curve or No-Load Curve which is also required to give the relation between field current and nominal E.M.F.; such a curve for a small three-phase machine is given in Fig. 44. The machine is run at its proper speed which must be kept quite constant, and simultaneous readings are taken of the armature pressure E and the field current, J. This curve should be determined throughout its whole range down to zero field current; in general there is a small amount of permanent magnetism present, so that the curve does not pass through zero. The lower part, however, is almost absolutely linear, and can be produced so as to cut the field current axis, and the point of section can be taken as the zero for that current so that the scale of field current is set off from that point: this saves complication in the use of the curve. It is best to draw the curve for armature pressure rather than line

pressure in a polyphase machine in order to avoid the chance of errors.

Although the saturation curve most usually employed connects the E.M.F. on open circuit, that is to say, the nominal E.M.F., with the corresponding field current J, it is sometimes more convenient to plot the E.M.F. against either the total field current-turns or polar current-turns. Further, since the speed is constant, the ordinates also give to some scale the value of the flux that has crossed the air-gap and is utilized by the armature, though the

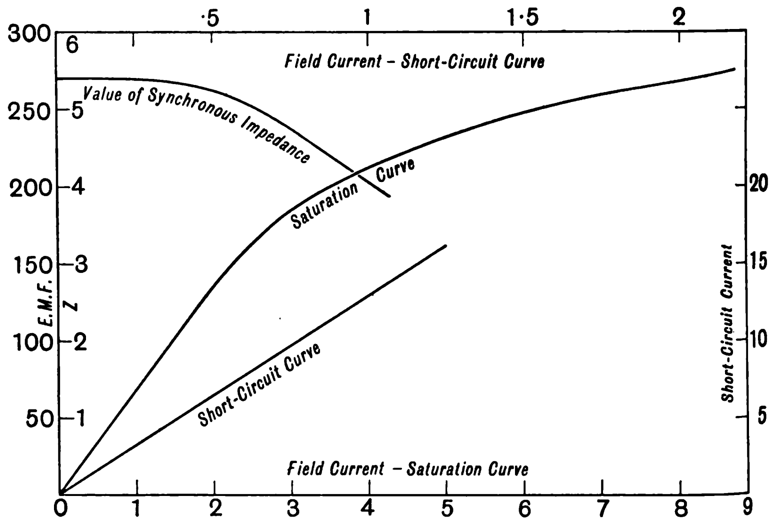


Fig. 44

total flux carried by the field magnet is larger than this owing to the dispersion or leakage from pole to pole.

Variation of the dispersion coefficient is indeed one of the great causes of difficulty in dealing adequately with the whole subject. It was indicated on p. 53 that it might be necessary to increase the exciting current above the value required to produce a given E.M.F. in order to provide a balancing current in the field to annul the armature's effect, and it will be well to consider this matter briefly before proceeding further, as it is one of much importance.

Suppose that a certain E.M.F. is existent on open circuit demanding a certain exciting field current. Since the field magnet circuit will experience a stray flux like that of any dynamo, this

excitation will be accompanied by its appropriate leakage, and the magnet's whole magnetic circuit will possess a definite Dispersion Coefficient q which depends on its form, etc. and can be found approximately in several well-known ways. Thus the excitation is concerned both with the useful air-gap flux producing the nominal E.M.F. and with a leakage from the field. But when we have to increase the excitation in order to balance the demagnetising effect of the armature, and thus restore the E.M.F. to its original value, an additional small increase in excitation being also demanded, as we shall see, in order to balance the effect of the cross-turns in altering the permeability of the pole faces, this increased excitation will necessarily further increase the flux straying from the poles and the rest of the field magnet circuit, which again means diminished permeability of the whole and further increase in excitation until a new stable condition is attained with a new value of "q." We may indeed express the matter as follows: the excitation required to produce a given nominal E.M.F., apart altogether from the necessary addition to balance the armature's effect, will depend on the form of the field magnet's circuit, its permeability and the change of dispersion coefficient with excitation. The matter may be illustrated as follows: In Fig. 45 is given the saturation curve of the small dynamo referred to on p. 55. The ordinates of the curve can be taken to represent either the E.M.F. produced at the given speed, 750 R.P.M., or the values of the air-gap flux to some other scale. Let us determine very carefully by test and calculation or both the value of the dispersion coefficient of the field magnet circuit say at the point L. It is taken as being 1·23; produce the ordinate LM to N making MN equal to 0·23 of LM. Then since the ordinates represent the air-gap flux, the length LN will measure the flux in the field magnet to the same scale. But we may take the reluctance of the

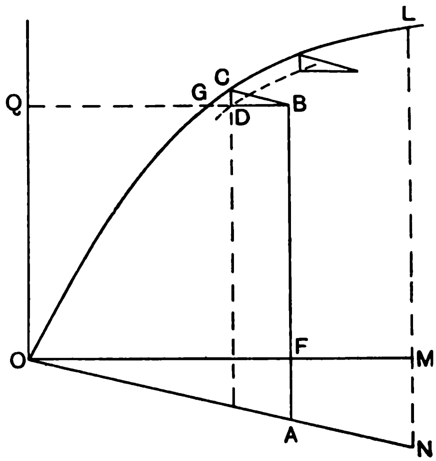


Fig. 45 (*abscissa* Field Current, *ordinate* E.M.F.)

stray flux paths of the magnet as being very nearly constant, although they will vary a little in distribution and form with change of total field flux. Draw the line *ON*, then it will follow that while the ordinates of the curve give the ***air-gap*** flux, the lengths measured from *ON* up to the curve will give the corresponding total ***field magnet*** flux, in fact the coefficient of dispersion is the ratio of any such lines as *LN* and *LM*. Let the condition of load be such as to require a nominal E.M.F. of 200 volts given by the point *Q*, and a magnet field current given by *QB*: this current very nearly corresponds to the actual condition of full load current with a 90° lag. To produce the necessary E.M.F. will require the flux given by *FB*, while that which is straying is then given by *AF*, so that the magnet must provide a total flux given by *AB*. But the relation between the field current and field flux is given by the intercept between the curve and the line *ON*, hence we must see where the line *AB* will fit in between the two. This is readily found by drawing *BC* parallel to *ON*; if the perpendicular *CD* be drawn to *QB*, it follows that we may take the field current *QD* as being required to give the nominal E.M.F., *BD* as that required to balance the armature effect, and the part *GD* as being the extra field current required to allow for the dispersion effects being different on load from those on no load. If one can assume that the demagnetising effect of the armature is constant for all excitations, then the saturation curve, corrected for dispersion corresponding to the assigned state of load current, and its phase can be traced out by moving the triangle *BCD* along the curve, so that *D* will trace out the proper saturation curve to use. It will be seen that each condition of load, whether as to magnitude of the current or as to its phase angle, will imply a different correction. The correction is thus so complex that it is generally merely estimated from experience. When using the curve for our present purpose, namely to determine the synchronous reactance, we can leave out of account the dispersion correction, and use the initial part of the saturation curve as actually found by test.

24. Short-Circuit Curve. The second curve required to determine X is the ordinary Short-Circuit curve of the armature and is found by short-circuiting the same through a low resistance ammeter and observing the simultaneous values of the field magnet current J_s and the armature current I_s for different field currents

up to that required to circulate at least the full load current in each of the simultaneously short-circuited armatures. The ammeter drop can usually be neglected if care is taken to use heavy mains for connection. The curve connecting the short-circuit current (which is best reduced to the actual armature current from the phase values if the machine is meshed) and the corresponding field current gives when plotted the short-circuit curve as shown in Fig. 44; this curve is practically linear over the range for which it is taken and has a definite slope which in the above curve will be found to be such that I_s/J_s is 12·5. But over the same range of field current it will be found that most saturation curves are also straight; in the above saturation curve the initial part is given by $E/J = 69{\cdot}5$. But when working on short-circuit all the pressure is consumed inside the armature, and hence the initial value of the Synchronous Impedance will be found by taking the ratio of the slopes, that is it here has the value 69·5/12·5 or 5·5 for the first part of the saturation curve. This quantity is given generally by the ratio of the E.M.F. and short-circuit current for the same value of the field current and is plotted in Fig. 44 not only for the straight part of the saturation curve, but for the higher values, the short-circuit curve being still straight.

In all machines of fair size the test may be taken as giving the synchronous reactance directly, since the resistance of a modern machine's armature is too small to affect the calculation, and in what follows we will usually suppose that only the reactance has to be taken into account. It follows that the short-circuit test involves the existence of almost absolute quadrature between the current and the E.M.F. and consequently from p. 50 it further follows that the armatures' magnetic effect is along the magnet's axis or is purely demagnetising. Further, we saw that when a current was flowing in the armatures, part of the field magnet current, J_D, could be looked on as being required to balance the armatures' demagnetisation, and that the amount of field current so utilized was given by the equation $J_D T = \nabla WI$. It then follows that the field current in the short-circuit test being J_s and that required for the armatures' demagnetising action being J_D the difference or $J_a = J_s - J_D$ must be available for producing the pressure necessary to circulate the current against the true reactance of the armature, so we can always break up the field current in the test into two parts such that $J_s = J_a + J_D$.

It is of interest to apply these results to the small alternator whose curves are given on p. 56. To circulate the full load current of 11·5 amperes per armature required a pressure of 64 volts as deduced from the short-circuit curve which showed a field current $J_s = 0{\cdot}92$ ampere for that armature current, corresponding to a synchronous reactance of 5·55. A rough measure of the true reactance per armature gave the value $X_a = 2{\cdot}3$, so that the actual internal E.M.F. demanded on short-circuit was $26\frac{1}{2}$ volts; noting that the relation between field current and E.M.F. is given by $E = 69{\cdot}5J$ for the first part of the saturation curve, this gives $J_a = 0{\cdot}39$ ampere. The balancing field current can only be found by determining the value of ∇; from the nature of the armature winding and the form of the poles, this was calculated in a manner to be considered later and found to be 0·25; the field turns numbered 3584 and the peripheral wires in all the phases were 738, so that with an armature current of $11\frac{1}{2}$ amperes, the value of J_D is $\dfrac{{\cdot}25 \times 738 \times 11\frac{1}{2}}{3584}$ or 0·58. The sum of this and J_a is therefore 0·92, or the same as J_s. This agreement is partly fortuitous, as the degree of accuracy possible in the various determinations does not warrant an expectation of agreement within a few per cent.

We will now use the synchronous reactance hypothesis to develop some further results.

25. External Characteristic.

Suppose we now enquire, what must be the E.M.F. of the machine in order that it may deliver a certain current at given P.D. and power factor? This is practically the same as the question of the pressure relations in a transformer. For if a current I be supplied per phase at the pressure V and the phase angle λ (Fig. 46) we have merely to add to V the pressures RI and XI as shown to determine E the nominal E.M.F. necessary to maintain that terminal pressure V. Conversely, if the E.M.F. and the power factor of the load are given it is easy to deduce V. For we have

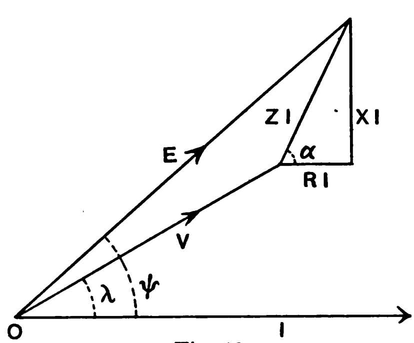


Fig. 46

it to take the projections of the vectors on the current vector and at right angles to it and we have

$$E^2 = (V \cos \lambda + RI)^2 + (V \sin \lambda + XI)^2,$$

which enables V to be found.

We will now use this method for finding the answer to the question:—how will the P.D. vary as a load of constant power factor increases, from zero upwards, when the exciting current, and with it the nominal E.M.F., remains fixed? On reference to Fig. 46 it will be seen that the question involves E and λ being constant, and we have then to find how V varies as I increases. We can most conveniently represent the current by the length of ZI since Z (the synchronous impedance) is constant; further the internal angle of lag α is constant being given by $\tan \alpha = X/R$; let us denote ZI by C, then C will be a measure of the current to a scale dependent on the value of Z. The triangle for EV and C gives

$$E^2 = V^2 + 2VC \cos (\alpha - \lambda) + C^2.$$

Writing β for $(\alpha - \lambda)$ we have

$$E^2 = V^2 + 2VC \cos \beta + C^2.$$

This is the equation to an ellipse lying obliquely and the usual transformation shows that the axes are respectively

$$\frac{E}{\sqrt{1 + \cos \beta}} \text{ and } \frac{E}{\sqrt{1 - \cos \beta}}.$$

The relation between V and C is called the External Characteristic of the dynamo under the assigned circumstances, and it follows that this characteristic, in terms of ZI for the current, will be the ellipse given above. Two limiting cases arise; if $\cos \beta = 1$ the ellipse reduces to a pair of parallel lines since one axis is infinite and the other is $E/\sqrt{2}$; this occurs when $(\alpha - \lambda) = 0$ or $\alpha = \lambda$, that is to say when the internal and external phase angles are equal. Since most dynamos have an internal angle of nearly 90°, this means the load is then fully inductive. The second case is when $\cos \beta = 0$, signifying that the axes are equal, and the curve then becomes a circle; this requires that $\alpha - \lambda = \pi/2$, or, with the abovementioned condition of large internal angle, that $\lambda = 0$, or that the load is non-inductive. Thus all lagging inductive characteristics should lie between the straight line and the circle of Fig. 47, each phase angle having its appropriate ellipse; ellipses outside the circle correspond to leading or condenser circuits, and it will be

seen that these involve a rise of pressure above that for non-inductive loading.

We see, then, that in its broad outline the theory of the synchronous reactance does account for the effect of the armature reaction in causing a rise of pressure on a leading load, and will be sufficiently accurate for general discussions of the questions that have to be dealt with; but on actual test neither of the limiting cases is found to hold good, both of them show curves above the deduced ones, the curve on 90° lag is not straight but is bowed upwards, and the non-inductive curve lies above the circle. The true characteristics of a dynamo for non-inductive loading and fully inductive loading are given in Fig. 48 for comparison with the deduced curves.

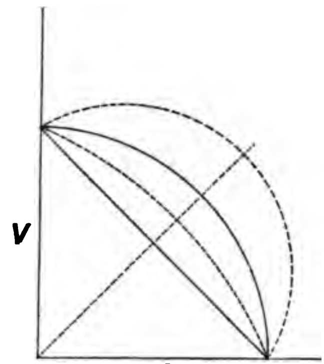


Fig. 47

It will be useful for the student to work out a few characteristics for the machine whose curves are given on p. 56 and the following additional data may be given. The machine was three-

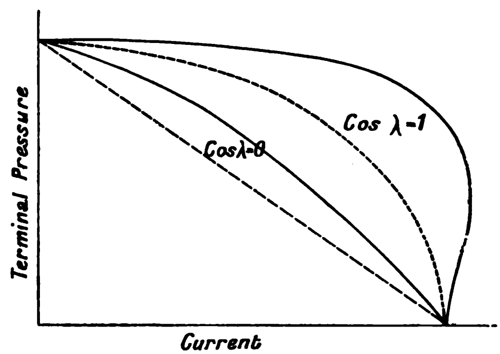


Fig. 48

phase with a mesh connected armature, the full load was 10 kilowatts at 200 volts and 25 periods, the speed being 750 R.P.M. The ohmic resistance per armature was 0·5 ohm, the field magnet carried 3584 turns, and the total peripheral armature wires counting all the phases were 738.

26. Determination of Exciting Current. One of our questions was that of finding the field current required to produce an assigned pressure with a given load, and the following approximate method might be used to solve this question. The given load will be represented by V and I in Fig. 46 set at the proper angle. Deduce as before, Fig. 46, the resulting nominal E.M.F. required and then from reference to the saturation curve read off the field current required to produce that E.M.F. If the pressure and current and their phase have been taken as those corresponding to the full load that the machine can carry, this current will be that required for the full load conditions. The field current for no load which will produce the same terminal pressure as that at full load is that current derived from the saturation curve that will give an E.M.F. equal to V. The intermediate conditions of load will be provided for by an adjustable resistance or rheostat in the field circuit, which is manipulated either by hand or by some suitable automatic device such as the Tirrell regulator, just as has been dealt with for direct current machines. This method of determining the range of exciting current is called the Synchronous Impedance method, and usually gives too restricted a range of field current, and the following method, known as the M.M.F. method, is preferred in practice for preliminary work. Let the load be given by the vectors OV and OI in Fig. 49. From the saturation curve find the magnetising current required for the production of the desired terminal pressure V and set it off along the vector OV as at OM. Now set off MN from M at right angles to the armature current, the value of the field current found from the short-circuit test, as that which is necessary to circulate the assigned armature current. We may then take ON as giving the necessary field current for the load. Strictly speaking this method is purely empirical, and is used for its simplicity and proved utility as an approximation. When two M.M.F.'s are actually in a line as with fully inductive loads this addition is legitimate, but with any other relation between the E.M.F. and current, the two will be inclined at an angle as stated on p. 50. Magnetomotive forces, being scalars, strictly speaking

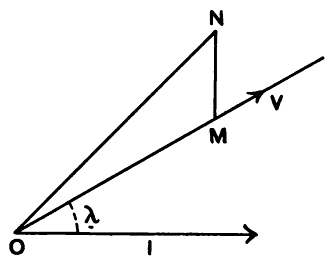


Fig. 49

cannot be combined as vector quantities, but here we may take them to be distributed more or less sinoidally round the armature, which indeed is very approximately the case with the cylindrical air-gap magnet we have supposed to be in use, and especially where such a magnet has distributed windings like that on the armature as is often the case in turbo-alternators, the approximation is very fair. We may therefore take it that it is then legitimate to combine the excitations as vectors.

For the present the above examples of the use of the synchronous reactance will suffice, as many others will occur as we proceed with our subject, and we may therefore take a further step and consider that we have succeeded in finding the two factors involved in the synchronous reactance, namely the true reactance X_a and the field current J_D equivalent to the effect of the reactance X_A as already done on p. 59. We can then derive a closer approximation to the solution of the problem of determining the excitation necessary for a given load as follows. Set off the pressure and current for the load as shown in Fig. 50 and then draw the vectors for the ohmic drop, RI, and the true reactance X_aI as shown. The nominal E.M.F. required is then given by OE. Along this line set off from the saturation curve the proper field current OM as in the other constructions, determine as already shown the field current appropriate to the load current and set it off from M at right angles to that current, the line ON will then give the proper field current to use.

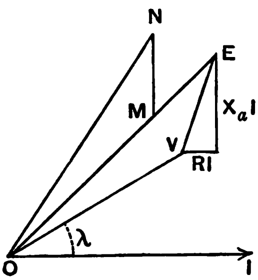


Fig. 50

27. Regulation. Another important question that can be dealt with by this construction is to enquire what is the alteration in pressure when a load is suddenly thrown off or on a machine running at constant speed with a definite excitation, which is such as to produce an assigned terminal pressure V in either case. If the machine is running loaded with that terminal pressure, the pressure will rise on throwing off that load by a certain percentage of the original pressure. Conversely if it be excited to give the same pressure on no load, the terminal pressure will fall by a different percentage on putting on the load. The percentage by

which the pressure rises on throwing off the load is called the "regulation up," while that by which it falls on throwing on the load is the "regulation down." The former is generally much smaller than the latter since the part of the excitation demanded for the armature effects is now available for producing flux; but the field current has to contend with a condition of less permeability inasmuch as most machines are designed to be fairly saturated in their full load condition. When the load is thrown on, the converse occurs; the demagnetising current diminishes that available for flux production, and as the saturation curve is straighter for

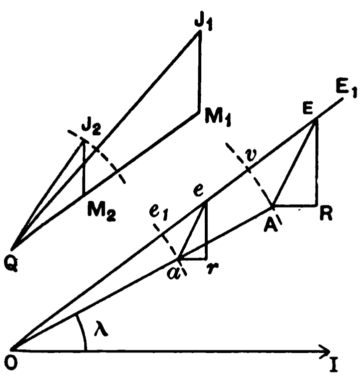


Fig. 51

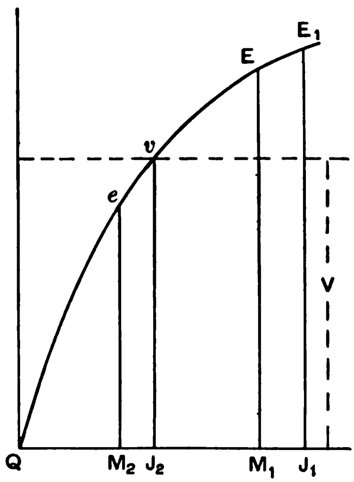


Fig. 51*a*

the lower part, a considerable fall in pressure must result. In general "regulation up" is what is specified, and the percentage rise is a measure of the "inherent regulation" of the machine. The production of machines with close inherent regulation is very costly, and it is common to permit a fair rise of pressure and instal an automatic field regulator to control the excitation.

The principle underlying the question can be dealt with as follows, giving an example of the last method of dealing with the armature effects. Let the load be such that a definite angle of lag λ is imposed and let the dynamo be running under full load, so that the P.D. V is given by OA in Fig. 51. Adding the ohmic drop AR and the true reactive drop RE we find the nominal E.M.F., OE. From Q we draw the corresponding field current QM_1 as derived

from the saturation curve, Fig. 51 *a*, and add on perpendicular t the current the field current to annul the armature effect, M_1J the applied field current is then QJ_1. If the load be suddenl taken off, this field current is operative solely in producing E.M.F and by reference to the saturation curve, Fig. 51*a*, the E.M.F. i seen to be J_1E_1. Set this off along OE and draw a circle through A as shown, then vE_1 is the rise of terminal pressure on throwing of the load, and its ratio to OA is the percentage "regulation up." For the other case it must be remembered that we start with th dynamo excited on no load to give the E.M.F. OA, which require a field current as derived from the saturation curve (Fig. 51*a*) o the value QJ_2. Now when loaded, this has to provide both th appropriate current to give the necessary nominal E.M.F. and als that required to balance the armature effect Since the load ha a constant impedance, the triangle Oea now connecting the P.D. E.M.F., and drop must be similar to the triangle OEA and ea indee fits in between the lines OA and OE. Further, the nominal E.M.F must agree with the appropriate field current on the saturatio curve, and the component of current for the field, J_2M_2, mus satisfy the ratio

$$AE/ae = M_1J_1/M_2J_2,$$

since the former fraction is the ratio of the armature currents The conditions can only be solved by trial and error, leading to a result which is approximately as shown. The triangle Oea is th new pressure triangle satisfying the conditions when the nomina E.M.F. Oe is produced by the field current QM_2 (Fig. 51*a*) and th balancing current J_2M_2 is in the proper ratio to J_1M_1 given above It will be seen that the regulation down is therefore given by ve_1 The noticeable point is the large fall ve_1 compared with the ris vE_1; in a practical case the other drops, namely ee_1 and vE, woul be far more nearly equal, and hence the up regulation is essentiall far closer than the down.

28. Load Curves. Another useful set of curves which can b immediately derived from the use of the resistance, true reactanc and J_D are those known as load curves. These connect the fiel current and terminal pressure of the machine for all conditions c loading both as to amount and as to phase. One of these has alread been dealt with, namely the Saturation Curve which is merely th no-load curve. Another can easily be deduced, for let OE, Fig. 5? be the saturation curve and E any point on it; with a 90° laggin

load the pressure will fall by an amount ZI, where Z is the true impedance of the armature and I is the full load armature current, which is generally taken for the test; it is usually near enough to consider the reactance only, so that if we set off ED equal to the drop IX_a, eD is the resulting P.D. But the excitation required to balance the armature in the condition of 90° phase is always in line with that for the E.M.F., and is here constant for all excitations since the load current is constant, so that if we draw DV to represent this field current, V will be a point on the full load 90° curve. Under such conditions it will be seen that it can be described by simply sliding the saturation curve as a whole in the direction EV. The curve can be found by test by putting a balanced load of air-cored reactances on the machine, or loading it in some equivalent manner; for various values of this load we adjust the excitation so that the assigned current is flowing in each phase, and then measure the P.D. With many machines the second curve is found to be non-reproducible by sliding the first; this is due to the fact that we have derived it from a no-load curve in which no allowance has been made for the extra excitation required by the increased dispersion referred to on p. 56. With some high speed machines with cylindrical air-gaps the agreement is quite fair.

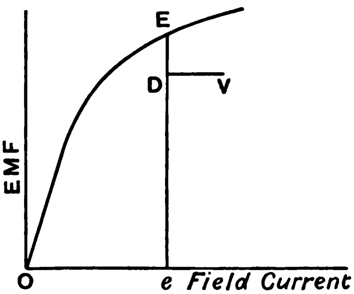


Fig. 52

Load curves between those for no load and 90° lag may be found as follows: each is supposed to be taken at full load current, so that in Fig. 50, p. 64, the total drop ZI and the current J_D will be constant. Hence if a set of diagrams like those shown in Fig. 50 are drawn for different assumed values of V, the corresponding values of the field current can be found very simply. These being plotted against the values of the P.D. give the required curve. The student should deduce such a curve for the small alternator whose data have been given, assuming a power factor of 0·8 both lagging and leading.

29. Separation of Reactances. It is appropriate to introduce at this point a brief account of certain methods for separating the true reactance from the demagnetising effect of an armature,

though they are all rather approximate owing to the uncertainty produced on load by the increased polar dispersion. Suppose we have determined the saturation and 90° full load curves we can find the true reactance as follows. If the curves are related in the proper way, namely that the curve Vv can be produced by sliding a particular triangle EDV parallel to itself, when E is on one curve V will be on the other. This fact must also be true when the triangle is moved down to the axis as at dev. But all saturation curves are straight at the origin, so that if dev is a fixed triangle, so is Oev, hence this must slide with the other. It follows that to find the unknown direction of slide, EV, of the original triangle we may proceed as follows: from the point V on the 90° curve draw the line VQ equal to Ov and then through Q draw a parallel to Oe; this will determine E, and as we saw that DE was the drop due to the true reactance, the value of ED divided by the current for which the 90° curve was taken will give X_a. Further it also follows that DV is the excitation, either in current J_D or current-turns, which is demanded for the armature's demagnetising effect with the same armature current. It may be noted that it is not essential to have the full curve Vv, for Ov is merely the excitation required to drive the selected current on short-circuit, a quantity already found in the short-circuit test*.

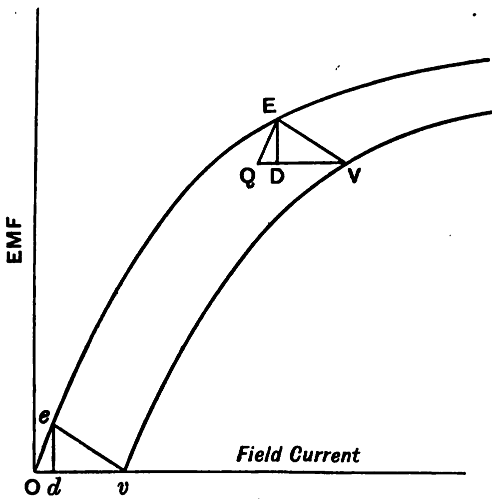


Fig. 53

Another very neat device is due to Kapp, and is especially applicable to the common case of star-wound three-phase alternators. It consists in making two short-circuit tests, one as usual with all the phases short-circuited, the other with only one so dealt with.

Let the lines connecting field current J and armature current I

* See also Warren, "Electrician," 1921, pp. 581, 654.

be as in Fig. 54 where $O1$ is the line for the single shorted phase, $O3$ for the three phases. Select the maximum load current and deduce the corresponding field currents J_1 and J_3. Each involves two parts, one being the current to produce the impelling E.M.F. sending the armature current I through the true reactance X_a of the armature, the resistance being as usual neglected. If we suppose that the reactance is not affected by the number of phases shorted, the excitation for the pressure IX_a will be the same in both tests though it is so far unknown; let it be J_a: then the current in the field available for balancing the armature demagnetising action with one phase is given by $J_1 - J_a$ and with the three phases by $J_3 - J_a$. But the latter should require very nearly three times as much field current as the former, so we have

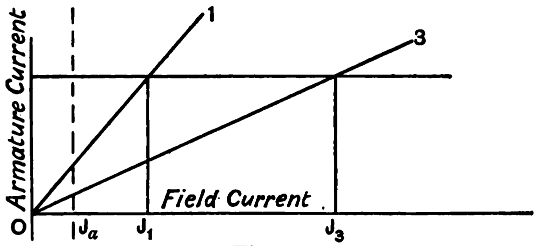


Fig. 54

$$J_3 - J_a = 3\,(J_1 - J_a),$$

from which J_a is given by $\frac{1}{2}\,(3J_1 - J_3)$. But from the saturation curve we know the E.M.F., say E_0, corresponding to J_a and hence the true reactance is given by $X_a = E_0/I$. Further, the field current demanded by the armature current I being J_D, that current is also deduced, being given by $J_3 - J_a$, and hence the two principal data are found. The method is of more doubtful applicability to mesh connected machines, as certain further assumptions have to be made.

The following pages include the more detailed investigation which may well be omitted by those who do not require it.

30. Gliding Fields. On p. 47 it was shown that provided the original M.M.F. curve for the armature was sinoidal, the resulting effect with a polyphase machine was to give an absolutely stationary band whose maximum was equal to that of either component with a two-phase armature, and $1\frac{1}{2}$ times that maximum with three phases; further, we saw that the maximum ordinate of the sine was given by the same expression, namely $\frac{1}{2\sqrt{2}}\,WI$, provided we measure the magnetic effect due to all the armatures in

terms of the virtual current in any one and the total peripheral wires in all the phases. This is sufficient for most purposes, but it is desirable to give a fuller enquiry into the matter, for those students who wish to go somewhat further; others can with advantage omit from here to p. 80. The results that will be obtained will be required when dealing later on with polyphase induction motors, as well as for the present purpose.

The actual distribution of M.M.F. round an armature cannot be sinoidal, but may have many and diverse forms some of which were briefly indicated in Figs. 32 to 36. The first step is to consider the representation of any such curve by means of a set of sinoidal curves of different orders, in fact to find the method of breaking up the given space distribution into the series of constituent harmonics, just as we did with the field form of the magnet. To save symbols we will use the single letter M to denote the maximum of any band, so that for our original sinoidal band M will stand for $\frac{1}{2\sqrt{2}} IW$. As before we will reckon distances in radians from the place where the field is zero, so that if m denotes the ordinate at any point and ξ the angular distance to that point, we must have

$$m = \sum_1^\infty M_q \sin q\xi,$$

where q denotes any of the odd whole numbers, M_q the maximum of the corresponding harmonic; if we wish to express the angle in terms of measured length x on the armature circumference, we can write $\xi = \pi x/a$, where a is the pitch of the distribution, which is the same as the polar pitch of the machine. We must now remember that the armature is not carrying a constant steady current but one which is itself alternating, and this current may also have time harmonics. Each will produce its individual effect on the belt of M.M.F., resulting in a variation of the appropriate M_q according to a similar series of sines, but for a time summation instead of a space one. It follows that we must write

$$M_q = \sum_1^\infty M_q \sin q\theta,$$

where M_q is the maximum of any constituent harmonic due to the current curve, and θ is such that

$$\theta = 2\pi t/T = \omega . t.$$

It is best to take up the question in steps, and we will first

assume that the current flowing is a pure sinoidal function of the time, that is $M_q = M_q \sin \theta$, giving a value of m expressed by

$$\sum_1^\infty M_q \sin \theta \sin q\xi,$$

so that this gives the value of m at any instant of time and at any point round the armature reckoned from the selected starting point.

Suppose that we now supply a second winding arranged so as to give an exactly similar space distribution round the armature, but starting exactly 90° away from the first; further, let the current supplied be at exactly 90° in time to the current in the first armature; then to find the value of m at the same point ξ on the armature as before we must increase both angles by $\pi/2$. Hence the complete expression for the value of m at the selected point due to both armatures is

$$\sum_1^\infty M_q \left\{\sin \theta \sin q\xi + \sin\left(\theta + \frac{\pi}{2}\right) \sin q\left(\xi + \frac{\pi}{2}\right)\right\}.$$

Select the qth harmonic in this expression; the appropriate value of m for the selected harmonic is then given by

$$M_q \left\{\sin \theta \sin q\xi - \cos \theta \sin\left(q\xi + \frac{q\pi}{2}\right)\right\}.$$

Two cases arise; firstly let q be 1, 5, 9, 13, etc. and the expression reduces to $M_q \cos(\theta - q\xi)$, while when q is 3, 7, 11, 15, etc. it becomes $M_q \cos(\theta + q\xi)$. These expressions show that the selected harmonic field for the first set will preserve its maximum unaltered provided we have $\cos(\theta - q\xi) = 1$, that is

$$\theta - q\xi = 0 \quad \text{or} \quad \frac{d\theta}{dt} = q\frac{d\xi}{dt} \quad \text{or} \quad \frac{dx}{dt} = \frac{1}{q}\frac{2a}{T}.$$

Expressing this in words it follows that the harmonic fields whose order is $(4n + 1)$, where n is any integer including zero, will all glide round the surface unaltered in shape, being each a pure sinoidal curve, all moving in the same direction, but with speeds inversely as their order; the speed of glide is such that the fundamental harmonic just traverses the face of a polar pair in the periodic time, the others all glide more slowly behind it. On the other hand those harmonics whose order is given by $(4n - 1)$ will glide round in the opposite direction. Sometimes the gliding action is referred to as the angular velocity of the belt, and for the fundamental this is clearly the same as the frequency of supply. If the field be multipolar the angular velocity of the fundamental would be f/P.

The effect of time harmonics in the source of current, leading

to corresponding ones in the "m" curve, can be briefly considered. If one such as $M_q \sin qt$ is interacting with the fundamental sine-distribution of the field it will be seen that it amounts to the same thing as supplying current by means of a dynamo of q times the normal frequency, and hence will result in a rotating band whose speed is q times that due to the fundamental supply frequency. Again, suppose that we have the qth time harmonic interacting with one of the same order in the "m" curve; this will result in a band given by the equation

$$M_q \left\{ \sin q\theta \sin q\xi + \sin q\left(\theta + \frac{\pi}{2}\right) \sin q\left(\xi + \frac{\pi}{2}\right) \right\},$$

which shows that it will have the same velocity as the fundamental band, forwards or backwards. Each time harmonic acting with any space harmonic will give a resultant band having a special speed. The really important member of the time harmonics is, however, the fundamental, and except in special cases no other need be considered. Further, all the belts will have velocities differing from that of the fundamental band, either greater or smaller, except for the very small bands due to those distributions which have the same order both in time and space harmonics, which will glide round unchanged like the fundamental band.

It is a simple step from this to the three-phase armature, the space and time distributions differ equally by $2\pi/3$, from which we can immediately write down the combined effect of the three fields when supplied with a pure sinoidal current; the effect of the qth harmonic in the m curve must be given by

$$M_q \left\{ \cos \theta \cos q\xi + \cos\left(\theta + \frac{2\pi}{3}\right) \cos q\left(\xi + \frac{2\pi}{3}\right) \right.$$
$$\left. + \cos\left(\theta + \frac{4\pi}{3}\right) \cos q\left(\xi + \frac{4\pi}{3}\right) \right\}.$$

A simple piece of trigonometrical manipulation leads to the following results: if q is any multiple of 3 the whole is identically zero, which is indeed evident for physical reasons; if q is one of the series 1, 7, 13, etc. or $(6n + 1)$, where n is any integer including zero, the solution is $3/2\, M_q \cos(\theta - q\xi)$, while if q is 5, 11, 17, etc. or $(6n - 1)$ the solution is $3/2 M_q \cos(\theta + q\xi)$. It follows that the fundamental curve will have $1\frac{1}{2}$ times the magnitude of either component and will glide round at a speed such as to pass a pair of poles in the periodic time, while all the others will glide round at slower speeds either backwards or forwards. Considerations similar to

those in the two-phase machine apply to the other harmonics both in space and time, so that we will still have those of equal order in both gliding at the same speed as the fundamental, but from the highness of the first order of harmonic that is possible and the smallness of the higher order amplitudes, all these are practically negligible. Hence we once more arrive at a sinoidal M.M.F. band gliding round the armature at fundamental speed with accompanying trains of other bands gliding round at different speeds.

31. The Fixed Armature Belt. So far we have referred the motion of the gliders to the surface of the armature and have considered only the relation between the resulting pure sinoidal bands and that surface. When the armature is supplied from outside the speed of glide is imposed by the impressed periodicity and this state of affairs will be considered when induction motors come under view; but in a dynamo the only way the currents are produced is by the rotation of the armature itself relative to the field magnet, or of the magnet relative to the armature. To fix one's ideas consider the first arrangement, the ordinary one in low speed multipolars which have fixed fields and rotating armatures. From our elementary consideration of the sinoidal M.M.F. curve on p. 44 it will be seen that the rotation of the armature is in such a direction as to bring the fundamental sinoidal band to rest in space and that one only, the others will glide round in space at various speeds. For example, the two-phase machine has a field of order 5 gliding round the same way as the fundamental so that its velocity relative to the fixed pole will be 4/5ths that of the fundamental; similarly there is a third harmonic gliding backwards relative to the first on the armature surface, which will have a velocity of 4/3rds that of the fundamental and so on. But still the only one that is quite stationary in space is the fundamental sinoidal band.

No apology is needed for this special insistence on the importance of fully realizing the objectivity of the fixed fundamental sinoidal band, as the concept is of primary importance.

When the dynamo has a rotating field and fixed armature, we must remember that the resulting band now glides round that armature, but is absolutely fixed in space relative to the poles for any specified condition of load.

From this it follows that our previous work on the magnetic effect of the armature band still holds good, provided that we take

in each case the appropriate value of the maximum of the fundamental sine which pertains to the Fourier expansion for the special form of winding which each armature carries. If we can find that maximum, we can still use the formula in the form $\frac{1}{2\sqrt{2}}WI$. As an example we will consider an armature with a single tooth per pole, that is with two slots, as shown in Fig. 55. This will produce a distribution of M.M.F. as shown by the figured rectangle whose height will be say WI. We have then to determine the value of the amplitude of the fundamental of this curve; let it be specified by the breadth of the tooth, say b' in the polar pitch a. On reference to the figure and to the expression given on p. 20 it will be seen that this corresponds to taking $\beta = 0$, and that consequently

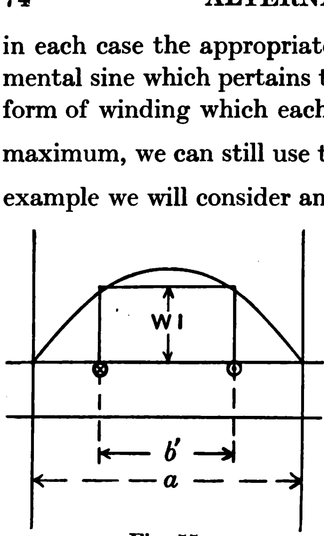


Fig. 55

$$\alpha = \frac{\pi}{2} - \gamma \text{ while } \gamma = \frac{\pi b'}{2a},$$

also for Y we have WI. On substitution it will be found that the amplitude of the fundamental sine is given by

$$\frac{4}{\pi} \sin \frac{\pi b'}{2a} WI.$$

Hence for this particular type of armature coil we must use the expression

$$\frac{1}{2\sqrt{2}} \frac{4}{\pi} \sin \frac{\pi b'}{2a} WI,$$

for calculating the maximum armature ampere-turns. For other forms of winding there will be other multipliers, for example with full pitch concentrated windings (where $b' = a$) the factor is $\frac{4}{\pi} \times \frac{1}{2\sqrt{2}}$, but as this is the classical form giving the well-known Guilbert coefficients, we will use this one as an example.

So far nothing has been said as to the nature of the surrounding field magnet; if it be of the cylindrical form as has been hitherto implicitly assumed, this new expression simply takes the place of the former one in all our calculations such as those dealing with the armature ampere-turns and the consequent turns on the fields, but we can now go a step further and derive expressions appropriate to machines with salient poles or non-cylindrical air-gaps.

There are two standard positions in which the sinoidal band of M.M.F. can stand in relation to the field, as shown in Fig. 56. One when it is coaxial with the salient pole, the other when it is at right angles to that position; the former corresponds to a current wholly at 90° to the nominal E.M.F., the latter to the condition of complete co-phasedness. The next step is to see how the magnetic effect of the band on the field magnet depends on those two positions. Consider the coaxial position, Fig. 56*a*, and denote the maximum

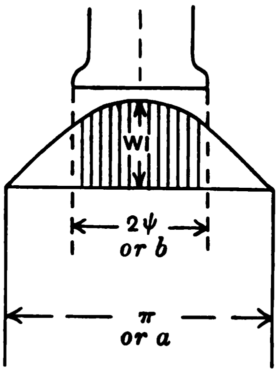


Fig. 56*a*

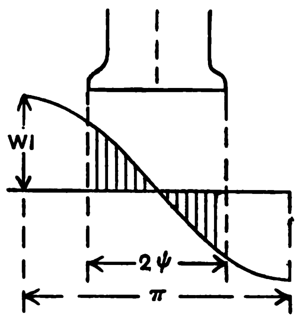


Fig. 56*b*

value again by M. Further let the polar pitch be as usual π and the portion of the pole covering the sinoidal band be 2ψ, so that if we denote the pole pitch by a and its breadth by b we have $\psi = \pi b/2a$. It will be seen that the mean effect of the portion of the band under the pole is given by

$$\frac{M}{2\psi}\int_{-\psi}^{+\psi}\cos\theta d\theta, \text{ or is } M\sin\psi/\psi.$$

It follows that in place of the simple expression hitherto used for the demagnetising armature effect, we must multiply by $\frac{2a}{\pi b}\sin\frac{\pi b}{2a}$. This effect corresponds exactly to the former constant dealing with the armature's reaction with a cylindrical gap, but we must replace the simple formula $\left(\frac{1}{2\sqrt{2}}WI = M\right)$ there used by a far more complex one, which with the one-toothed armature leads to

$$\frac{1}{2\sqrt{2}}\frac{4}{\pi}\sin\frac{\pi b'}{2a}\frac{2a}{\pi b}\sin\frac{\pi b}{2a}WI \quad \text{or} \quad \frac{2\sqrt{2}}{\pi^2}\frac{a}{b}\sin\frac{\pi b'}{2a}\sin\frac{\pi b}{2a}WI.$$

The coefficient of WI is the first Guilbert constant for the particular winding; as it will vary for others we will denote it in the generalized case by the letter Δ, so that the demagnetising ampere-turns are now always given by ΔWI.

In the simplified treatment leading to the synchronous reactance concept it was stated that we could either deal directly with the armature's demagnetising turns as given by the simpler formula, or suppose that the band of magnetic force was effective in producing flux, when it could be replaced by an extra reactance X_A in the armature over and above the true reactance X_a. Similarly we can look on this new belt either as being dealt with by the field magnet, the most helpful method, or as being equivalent to a reactance which we will denote by X_Δ to distinguish it from the former. But this reactance, or the equivalent magnetic belt, has one property in which it differs from the old one; the latter was associated with the whole armature current and produced a magnetic effect at right angles to the current band; but the present magnetic effect is tied on to that part of the current which is in quadrature with the nominal E.M.F., not to the whole current; it follows that when we take the current's phase into consideration the magnetic effect of the band must be given by $\Delta WI \sin \phi$ and the corresponding E.M.F., if we replace it by a reactance, must be expressed by an E.M.F. given by $X_\Delta I \sin \phi$, both being at right angles to the quadrature component of the current.

32. The Cross Reactance. We must now consider the second standard position, namely that at right angles to the main field. This produces a very different effect; the armature magnetic band can now force flux in a local circuit up one-half of the polar face and down the other, in fact it produces a true cross-magnetising flux just like that in the direct current dynamo and with similar consequences, since the band is quite fixed in space, and thus causes the weakening and strengthening of the polar horns that give so much trouble in dealing with direct current dynamo calculations. The physical effect is, as it were, to shear the flux along the pole, so that the maximum no longer occurs exactly at mid-pole but to one side, that is to say, we can no longer assume that the maximum nominal E.M.F. occurs at mid-pole. If the effect were a simple shear we could say that the maximum of the field form curve was displaced from its central position by a small angle β, and that in

consequence we slightly diminished its maximum effective value in proportion to cos β. This is a useful approximation, but the effects of the alterations of polar flux are more disturbing. It was mentioned on p. 57 that one effect of the necessity of providing turns on the field to balance the armature was to alter the dispersion coefficient of the magnet in a manner which depended on the load. The same will also apply here, but in addition to the increase necessitated by balancing the direct action of the demagnetising turns, we must provide some excitation to keep the air-gap flux up to the required total since the field magnets are usually fairly saturated, and consequently the gain of flux-bearing power experienced by the weaker horn in no way counterpoises the loss of that power at the strengthened horn. The result is to render the exact form of the saturation curve (see p. 56) that has to be used in our load calculations still more indeterminate.

When in the position of Fig. 56*b* it will be seen that the mean M.M.F. due to the armature is given by

$$\frac{M}{\psi}\int_0^{\psi}\sin\theta d\theta,\text{ or }\frac{M}{\psi}(1-\cos\psi),\text{ that is by }M\frac{2a}{b}\left(1-\cos\frac{\pi b}{2a}\right),$$

so that when we insert the proper expressions we arrive at the second Guilbert constant, namely

$$\frac{2\sqrt{2}}{\pi^2}\frac{a}{b}\sin\frac{\pi b'}{2a}\left(1-\cos\frac{\pi b}{2a}\right).$$

As before we will use a special letter Γ to denote the value of the constant term appropriate to other cases than the actual one to which the above formula applies, so that the cross-magnetising ampere-turns $= \Gamma WI$.

The actual range of values of the two constants in ordinary machines is for Δ from about 0·4 to 0·21, and for Γ from about 0·28 to 0·12.

One important matter must be mentioned; it will be noticed that the ratio Γ/Δ is tan $\pi b/4a$ and is therefore quite independent of the nature of the winding of the armature but depends solely on the polar breadth relative to its pitch.

The cross-magnetising effect, unlike the demagnetising one, cannot be compensated by an opposing field winding; it virtually produces a definite flux superposed on the main flux under all conditions, as is indicated by the necessary resulting shift in the phase of the nominal E.M.F. It follows that it can best be repre-

sented by a reactance. The component of current to which it is tied is, however, the inphase one, since all cross-magnetising is due to that component. We will call this reactance the Cross Reactance and denote it by X_Γ, so that whenever we have a salient pole machine we must bear in mind the presence of a new E.M.F. which will be given in amount by $X_\Gamma I \cos\psi$ and is at right angles in phase to the inphase component of the current. The actual determination of the value of this reactance is very difficult and complex; a method that has been used with great success is to calculate very carefully from the armature windings, the gap and wire distribution the actual flux produced by the armature, when in the cross-magnetising position, for one ampere in its circuit. From this curve of gap-flux due to the armature, the corresponding E.M.F. can be found exactly as one finds the E.M.F. of an ordinary field distribution, for it is equally fixed in space; it follows that knowing the E.M.F. demanded by the unit current we can calculate the value of X_Γ. But this procedure is far too complex for ordinary work, and an approximate method must be adopted; this is best illustrated by reference to the results obtained with the small alternator mentioned on p. 56. We there found that the synchronous reactance was 5·55 while the true reactance was 2·3; it follows that the reactance equivalent to the armature's effect on short-circuit, that is its full demagnetising effect, was represented by the difference, or 3·2, and this may be taken as the appropriate value of X_Δ for the present consideration. Now if we neglect saturation effects, it is approximately accurate to take the ratio of X_Γ to X_Δ to be the same as that of Γ to Δ. But the latter ratio is known to be $\tan \pi b/4a$; the present dynamo had a polar pitch of $23\frac{1}{2}$ cm. and a polar breadth of $17\frac{1}{2}$ cm. from which we deduce the value of the ratio as being 0·66, so that the approximate value of the cross-reactance for the dynamo is 2·1.

We must now see how we can utilize the four constants R, X_a, Δ and X_Γ to solve the problem of finding the proper excitation of a dynamo for a given load.

We start in the usual way by setting off the load vectors V and I at the assigned angle λ (Fig. 57), then add on the ohmic drop and the drop for the true reactance. OX would then be the position of the nominal E.M.F. in the original simplified case, but here it is shifted forward by a small unknown angle β such that if we drop the perpendicular XE on the new line, OE is the proper

nominal E.M.F. in phase and magnitude. It will be seen that this means that another E.M.F., namely EX, has appeared, and this is the one due to the cross-reactance; for the figure shows that it is at right angles to the inphase current component; it follows that although we do not know the angle β and consequently are equally ignorant about ψ, we do know that the length of EX should be $X_T I \cos\psi$, if the figure is correctly drawn; fortunately a simple solution is possible. Produce RX and OE to meet at e, then the angle eXE is also ψ, and it follows that eX must be $X_T I$. We can now reverse the construction, thus, set off the figure up to RX as before, then produce RX by the known amount $X_T I$, join Oe, and drop a perpendicular from X on Oe; the length OE is the required nominal E.M.F., and the angle ψ is found.

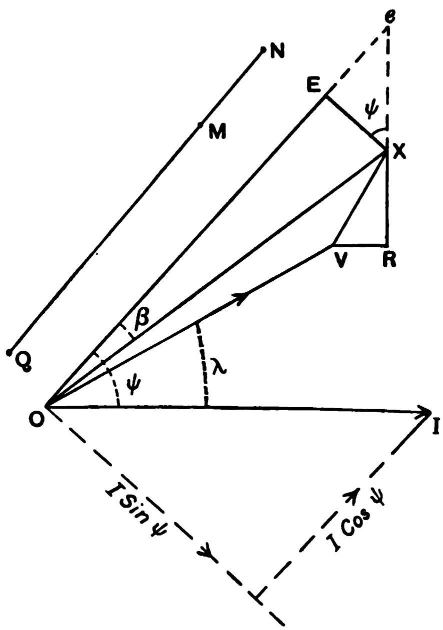


Fig. 57

The excitation is then determined by first setting off the length QM to give to scale the field current required to produce the E.M.F. given by OE; to find the extra current required to balance the demagnetising effect, we must remember that we are only concerned with the quadrature component of the armature current, so that the extra current must be added on in the same line as QM, perpendicular to I. Further, if the short-circuit test shows that the given current required a field current of the amount J_D to balance it, the current we have to add on the field is $J_D \sin\psi$; making MN of this value, we derive the proper exciting current QN for the given load. The student should apply this method to the small machine referred to above.

It may be worth while correlating this construction with the former one in which a single constant X_A was taken for all

the armature effects. A reference to Fig. 57 will show that if we assume a common value for the three coefficients so that $X_A = X_\Gamma = X_\Delta$, the triangle *EeX* has its sides respectively equal to $X_A I$, $X_A I \sin \psi$ and $X_A I \cos \psi$, so that the side *Ee* must represent the effect of the reactance X_A in respect to its demagnetisation, while *EX* represents its cross-magnetisation, hence the plain synchronous reactance method simply involves the omission of the line *EX* and a supposition that all the reactance effects of the armatures are due to similar distributions of magnetising turns.

33. Parallel Operation. Up to the present we have considered an isolated dynamo delivering a constant load to an outside

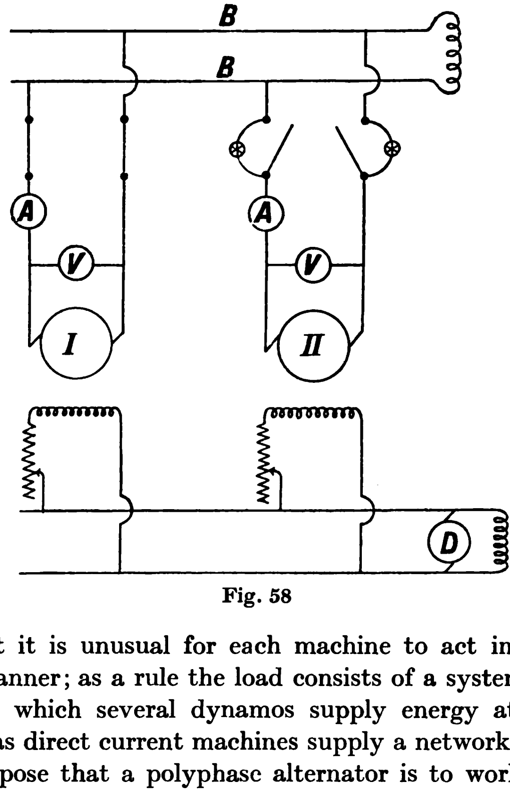


Fig. 58

circuit, but it is unusual for each machine to act in this independent manner; as a rule the load consists of a system of supply mains into which several dynamos supply energy at the same time, just as direct current machines supply a network in parallel. Let us suppose that a polyphase alternator is to work on mains between which the pressure is maintained constant at some value V and frequency f. One condition that must be fulfilled is that it shall rotate at such a speed as to produce the same frequency,

so that if its R.P.S. be N and its pole pairs P we have $f = PN$. The other condition is that at every instant the pressure at its terminals shall be equal and opposite to that between the mains. The first thing is to see how this arrangement can be brought about.

Let the frequency in the mains in Fig. 58 be that given by $\omega = 2\pi n$, and suppose we are running the incoming machine at some other frequency corresponding to ω_1 but with the same E.M.F. Let one terminal and one main be joined and place a lamp across

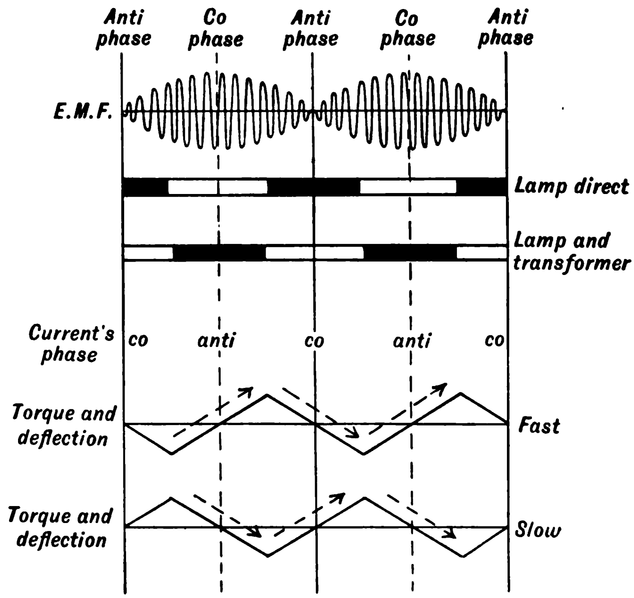


Fig. 59

the open gap between the other terminal and main. The pressure to which that lamp will be subjected is $E_1 \sin \omega t + E_1 \sin \omega_1 t$,

$$\text{or} \quad 2E_1 \cos \frac{\omega - \omega_1}{2} t \left(\sin \frac{\omega + \omega_1}{2} t\right),$$

that is to say, it varies with the time according to the law $\sin \frac{\omega + \omega_1}{2} t$ or with a periodic time $T_1 = \frac{4\pi}{\omega + \omega_1}$ while the amplitude varies harmonically with the periodic time $T_2 = 4\pi/\omega - \omega_1$. As shown in Fig. 59 there will necessarily be zero lamp P.D. at the moment of anti-phase and a maximum P.D. when the E.M.F.'s are in phase and it is at the first we have to aim before making the

circuit complete. It will be seen that unless the periodic time T_2 of the variation of the maxima is reasonably large, the lamp will not have time to cool in the interval near anti-phase, and will hence glow with average moderate brightness, but if the periods differ by only a small amount, which will be the case in practice since we must run at about the right speed to attain the desired E.M.F., we can write $\omega_1 = \omega - \delta\omega$, where $\delta\omega$ is a fraction of ω. The equation then becomes

$$2E_1 \cos \frac{\delta\omega}{2} t \sin \left(\omega - \frac{\delta\omega}{2}\right) t.$$

When the variations become observable $\delta\omega$ is only a few per cent. of ω, so that we practically have

$$2E_1 \cos \frac{\delta\omega}{2} t \sin \omega t,$$

the periodic time of the amplitude is $T_2 = 4\pi/\delta\omega$, and that of the component wave is nearly $2\pi/\omega$, that is nearly the same as the mains. Thus with 50 period main-frequency and the incoming machine 10 per cent. slow, we shall have about 20 seconds for the complete periodic time of the change of amplitude, or about ten seconds from maximum to minimum; in general we must adjust the dynamo's speed a good deal nearer than this to operate with any success.

The lamp being thus subjected to a periodic variation in P.D. will show alternate intervals of light and darkness, the duration of which will partly depend on its thermal flywheel constant; these are roughly indicated below the curves in Fig. 59. The condition of true anti-phase which must exist before we can close the open gap without danger is then indicated by the lamp staying steadily dark; if the lamp holds dark for a few seconds we can make the switch which closes the gap across which the lamp is connected.

This method is a very simple and useful one for laboratory work but it has two disadvantages, the first is that being a null method the darkness of the lamp may be due to its having failed; it would from this point of view be better to use the period of maximum brightness as the criterion of anti-phase: this can very easily be secured as follows. With an ordinary high pressure machine it is impossible to put the lamp directly across the gap; we should necessarily use a transformer with the primary across the gap and the lamp in its secondary; hence it is quite easy to arrange a transformer with two primaries across the mains and the dynamo terminals respectively, and with one secondary supplying the lamp

shown in Fig. 60; by properly connecting this transformer up will be seen that we can arrange that anti-phase pressures shall indicated by brightness instead of darkness, and this condition usually employed. The corresponding times of light and phase lation are then as shown in the lower "light" diagram of Fig. 59.

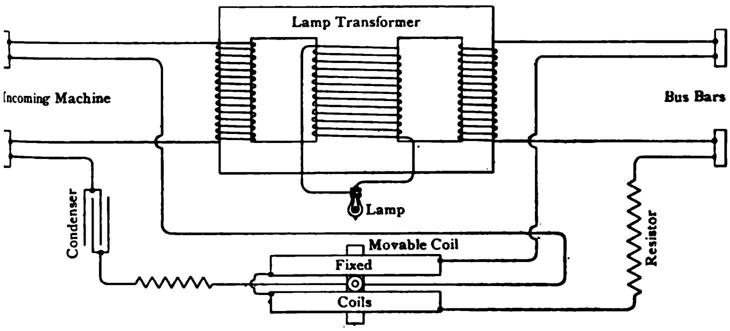


Fig. 60

ıt both the above suffer from the second disability; the method ves no criterion as to whether the machine is running too fast too slow, that is whether $\delta\omega$ is positive or negative.

34. **The Synchroscope.** There are several methods used to ercome this difficulty, most of which involve special connections the lamp-transformers. A simple one is as follows (Fig. 60 *a*); the

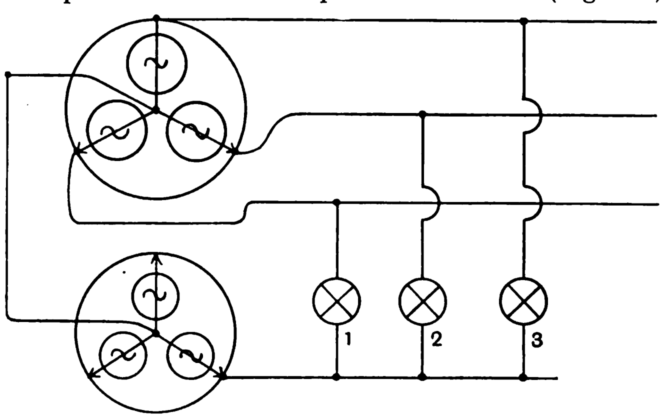


Fig. 60 *a*

nsformers and their lamps being simply placed across the mains l the machine as shown, it will be evident that if the dynamo is r wound, and if both it and the main system have their neutrals

earthed through the usual reactance coils, and therefore virtually joined together, when lamp (1) is dark and the others glow steadily, the machine must have the right phase and be running synchronously; any relative angular velocity will be shown by the lamps lighting up in succession, the sequence being an indication of the state of relative motion.

It is, however, more common to provide special instruments for this purpose which are called Synchroscopes; most of them depend on the utilization of induction-motors and consideration of them will therefore be deferred, but the following ingenious form, the Weston synchroscope, may serve to illustrate the use of such an instrument. Its essential part consists of two coils, a fixed one and a movable one, the latter with the usual spring control; it is just like a wattmeter except that both coils are wound with fine wire. Omitting for the moment any reference to the lamp circuit of Fig. 60, we see that the mains are connected to the fixed coil through a non-inductive resistance and the moving coil is in series with a condenser across the dynamo terminals. Suppose the latter is running in synchronism, then the current in the fixed coils will be in phase with the main pressure, that in the moving coil in quadrature with the dynamo pressure, and hence there can be no couple on the moving coil; and this is true whether the currents be anti-phase or in phase. The coil carries a pointer which shows through a hole in an opal glass front when the deflection is zero. We will only consider the case of fairly close synchronism having been attained. Since anti-phase and co-phase of the E.M.F.'s both result in zero couple on the coil, the maximum couple will occur between the two conditions of phase and will be in opposite directions on each side of the null points, so that the torque will vary in the way shown by the upper zigzag line as the resultant pressure follows the top curve, that is to say, the needle will swing to and fro from one end of the scale to the other as the phase changes: the direction of motion during each swing is shown by the arrows. The above may be taken as referring to the condition of the dynamo running faster than the synchronous state. Now let it run just below that speed or "slow," the rates of change of the E.M.F.'s with speed will be reversed and thus the directions of the torques will also be reversed, so that the torque zigzag line is then the lower one. Now if the needle were always visible it would tell us very little, since confusion must arise as to

ne meaning of the swings. This is avoided by connecting the lamp ı such a manner as to go dark at inphase and bright at the de- ired condition of anti-phase as shown in the figure. Hence as the eedle is only visible through the opal glass when the face is luminated from behind, only the swings on the "light" part of he diagram can be seen, so that when the needle moves across he dial from left to right it means "too fast," when from right to eft "too slow." Hence we can adjust the dynamo speed accord- ngly, the swings getting slower and slower till the needle just hows at the opening indicating permanent anti-phase, and then he switch can be made, as the two pressures are then exactly pposed, that between the mains being equal and opposite to that roduced by the dynamo. Strictly speaking this assumes that the wo waves have identically the same shape, and if they have not, arasitic currents largely of higher order than the fundamental vill circulate round the armatures; with machines which give a easonably close approximation to a pure sinoidal wave of E.M.F. his effect is practically negligible. But we must now consider what appens when the machine is so adjusted as to share in taking up load, that is to say, whether it will behave stably and respond to demand for increase of load without getting what is known as 'out of step," or without breaking away from its synchronous con- lition of rotation.

35. Synchronising Couple. Suppose that due to some nequality of turning moment on he dynamo it is caused to depart rom the steady running position o that the pressure vector shown y the lower line (Fig. 61) shifts y a very small angle σ correspond- ng to a physical shift α of armature elative to pole of the amount given y $\alpha P = \sigma$, where P is the number f pairs of poles, this shift being ither a lag behind the steady osition or a lead in front of it.

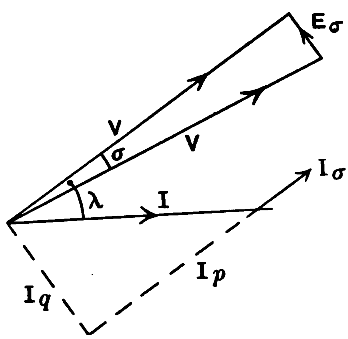


Fig. 61

mmediately there must be brought into existence a small pressure E_σ directed as shown in the figure, so that the pressure between e mains balances the sum of that due to the dynamo and this

new one. The value of the small pressure is very nearly σV, and it is almost at right angles to the new position of V. It will send a current through the armature of the machine and this current will lag nearly 90° behind the small pressure in any modern machine with high armature reactance. Hence this current will be practically in phase with the P.D. and thus extra work will be done by the supply mains tending to pull the machine back to its mid-position. Similarly if the dynamo leads, since E_σ is then reversed in direction, the current will oppose V and work will be thrown on the dynamo tending to bring it back into place. We will call this new current the synchronising current, I_σ. The method in which the current produces the necessary torque is exactly the same as torque production on the direct current machine. It is due to the stationary belt of flux produced by the armature, corresponding in every respect to the stationary magnetic field produced by the armature of an ordinary direct current machine; and just as the torque in the latter is due to the interaction of the main field and the armature field, so is the torque in the alternator. Now the main current consists of a power component and a quadrature one, and taking the position shown, the synchronising current will add to the former giving an extra power component of the value I_σ and nearly unaltered quadrature component. The effect of the synchronising current is therefore principally to alter the cross-flux. The question now arises, what is the relation between E_σ and I_σ? On p. 78 it was seen that the effect of the cross-flux was equivalent to a reactance X_Γ and it was also seen that the E.M.F. E_σ is at right angles to the current's power component or to I_σ very nearly if we neglect the small difference between the external and internal phase angles λ and ϕ. The current I_σ has in addition to cope with the true reactance of the armature, hence the relation required is given by $E_\sigma = I_\sigma (X_a + X_\Gamma)$. In fact the angle of displacement σ can be looked on as being made up of two parts one being due to the pressure $I_\sigma X_\Gamma$ which accounts for the shear of the flux across the face, the other $I_\sigma X_a$ due to the true armature reactance. We have seen that the ordinary treatment is to take X_Γ as being the same as the demagnetising reactance or X_Δ or to our old reactance X_A (p. 80) so that in this case we have $E_\sigma = I_\sigma (X_a + X_A)$, or since $X_a + X_A = X$ the synchronous reactance, $I_\sigma = E_\sigma / X$. But the synchronising power is VI_σ as V and I_σ are cophased, also the figure shows that $E_\sigma = \sigma V$, so that the synchronising power is

$\sigma V^2/X$. It is usually taken that we can write this in the form σVI_s, where I_s is the short-circuit current that would be impelled by full load pressure V, although such a large current would not actually ever flow.

The value of the synchronous impedance of the machine referred to on p. 56 is given in the same figure as the saturation curve and the short-circuit curve; it will be seen that it varies with the excitation, but it is the value found in this way that is generally used in the formula. This usually gives too low a result for the synchronising power as is evident from the former considerations, since we are crediting the machine's armature with too large an impedance. With salient pole machines the difference is considerable and can be allowed for by a more or less empirical relation between the synchronous impedance and that to be used in the formula, but the simple one given above is usually near enough for a first approximation, consequently we will in future assume that the synchronising power is given by σVI_s, where I_s is the short-circuit current for full load excitation. This power translates itself into an accelerating or decelerating shaft torque, and if the R.P.S. of the machine is N corresponding to an angular velocity $\Omega = 2\pi N$, the torque will then be $L = \sigma VI_s/\Omega$.

We can readily express this torque in terms of the actual angle α by which the armature has shifted in space instead of the angle σ in the vector diagram, since we have $\alpha P = \sigma$, so that $L = \alpha VI_sP/\Omega$. Further, we can write $L_0 = VI_sP/\Omega$, where L_0 is the torque per radian displacement, a definite constant of the machine, although such a relative displacement is usually impossible. It must be remembered that this torque is measured in electrical or Joule-Radian units and not in mechanical units. The relation expressed is that of equating (torque $\times$ angular velocity) to (volts $\times$ amperes) and the numerical relation between a torque expressed in electrical units and the equivalent expression either in kilogram-meter or foot pound pound foot units is readily found. For $L_0 = VI_sP/\Omega$ is of the dimension Joules, being (watts $\times$ time), and hence must be multiplied by 10^7 to reduce it to absolute units. Since the factor to reduce kilogram-meter units to absolute units is $10^5 \times 981$, it follows that if we wish to express L_0 in kilogram-meters we must divide by 9·81. Similarly we can express the moment of inertia, given in absolute units, in terms of kilogram-meter ones. The factor is 10^{-3} in respect to mass and 10^{-4} in

respect to (distance)^2 or 10^{-7} in all. This is the same factor that turns absolute units of energy into Joules, hence an equation in the former units is unaltered in practical units if the moment of inertia be expressed in kilogram-meters: this is a convenient point to remember. If the moment of inertia is required in metric-ton meter units the divisor is 10^{10}.

36. Free Swings. For elucidation we will retain the electrical units for couple and moment of inertia and reduce only when numerical results are required. The torque resulting from an assigned displacement will produce an acceleration of the moving parts, and if the moment of inertia is denoted by K, the equation of motion is

$$K\frac{d^2\alpha}{dt^2} + L_0\alpha = 0.$$

The result is then that the displacement varies with the time in a simple harmonic manner and the periodic time of the resulting oscillation about the mean position is given by $n = \frac{1}{2\pi}\sqrt{\frac{L_0}{K}}$. Sometimes the full expression $n = \frac{1}{2\pi}\sqrt{\frac{VI_sP}{\Omega K}}$ is more convenient, but a still more useful form is given by writing q for $2\pi n$ so that the natural swings of the system have a time of swing given by $q = \sqrt{L_0/K}$.

It must be noted that as written the expression refers to a single armature. If we wish to use it to apply to a three-phase machine then we must use $3VI_s$ for VI_s if the pressure and current refer to any one armature, or $\sqrt{3}VI_s$ if they refer to the line.

37. Forced Swings. The periodic motion of α about its mean will be either stable or unstable; it would take us too far just at present to deal with this point, but with reasonably small values of α and in the absence of special outside influences it may be taken to be stable, and will be gradually damped out by the ordinary mechanical frictional forces, or still more powerfully by special devices to be considered later on.

The above considerations apply to the natural swings of the system when free; but we may have impressed periodic couples acting, such for example as a periodic variation in the turning moment on the machine. Suppose that such a periodic disturbance exists having the frequency n_0 given by $p = 2\pi n_0$, and let the

magnitude of the disturbing couple be D. The equation of motion is now

$$K \frac{d^2\alpha}{dt^2} + L_0\alpha = D \sin pt.$$

Substituting q^2K for L_0 this gives

$$K \left(\frac{d^2\alpha}{dt^2} + q^2\alpha\right) = D \sin pt.$$

Since the coefficients of the right hand of the equation are constants, the angle α must vary periodically with the same frequency as the disturbing couple namely that given by $p = 2\pi n_0$. If A denote the maximum of the disturbance and if we note that the maximum of $d^2\alpha/dt^2$ must be $-Ap^2$, on equating the maxima of the two sides we obtain $K(q^2 - p^2)A = D$. But we know that $Kq^2 = L_0$, and hence if we write B for the ratio of the two couples D and L_0 so that B is the angular motion producible by D, we have $D = BKq^2$ and $\frac{A}{B} = \frac{q^2}{q^2 - p^2}$. The ratio of the two angles is called the "reaction quotient."

It will be seen that in default of any damping effects the resulting amplitude tends towards infinity with coincidence of natural and forced periods, and at that moment the phase of the motion changes by 180°. The condition indicates, then, a condition of unstable operation and we can use it to establish some useful expressions dealing with the critical value of the moment of inertia for the machine to fulfil stable operating conditions. The disturbing torque is usually one due to the prime-mover, and as all the machines we consider are direct driven, it is therefore some multiple or submultiple of the speed of the alternator which we have denoted by N. Hence if λ denote any such multiple or submultiple we must have $p = 2\pi\lambda N$ or $p = \lambda\Omega$. But we have $q^2 = \frac{VI_sP}{K\Omega}$ so that the critical condition being $q^2 = p^2$ the critical moment of inertia is given by $K = \frac{VI_sP}{\lambda^2\Omega^3}$. It therefore depends on the cube of the speed and on the square of the factor λ.

Before developing this expression in more detail we must consider the method by which the extra damping effect referred to above is provided.

38. Damping: Amortisseur. This is done by surrounding the whole armature with a set of thick copper rods lying parallel to

the machine's axis both in the interpolar space and through the pole faces, and so supported so as to form a complete "squirrel cage" round the whole armature; the two sets of ends are severally connected together by rings thus forming a completely short-circuited system. When the armature is running steadily, its magnetic field being fixed relative to the armature and of constant value, this system of short-circuited conductors will also be fixed in space relative to that field. But if any relative motion of the two takes place due to an alteration in the angle between field and armature such as a momentary change of torque produces, the armature flux band will sweep slowly across the bars and hence E.M.F.'s will be induced in them proportional to the rate of change of the angle of displacement and these E.M.F.'s will induce corresponding "eddy" currents in the rods. Usually the periodicity of the to and fro relative swing is low, and although the rods have very small resistance, the periodicity of the currents in them is too small to cause the reactance of the rods to be appreciable, hence we may take the current in a rod to be almost exactly in phase with the E.M.F. induced in it. The induced current will react with the armature field and produce a torque on the armature, and since the energy demanded by the rod currents has to come from the kinetic energy of the armature, this torque will oppose the change of configuration or will act as a damping torque; this system of rods is called a "damper" or "amortisseur," and we can express its effect by reference to the torque it would produce at unit relative angular velocity between the field magnet or amortisseur and the armature flux; this torque per radian per second will be denoted by F, so if the displacement at any moment has the value α the torque will be $F d\alpha/dt$.

The equation of motion now takes on the form

$$K\frac{d^2\alpha}{dt^2} + F\frac{d\alpha}{dt} + L_0\alpha = 0,$$

or putting q^2 for L_0/K, and m for F/K,

$$\frac{d^2\alpha}{dt^2} + m\frac{d\alpha}{dt} + q^2\alpha = 0,$$

the motion being free though now damped. This is the familiar equation whose solution is damped harmonic motion; that solution is given by

$$\alpha = A\epsilon^{-mt}\cos\left\{\left(q^2 - \frac{m^2}{4}\right)^{\frac{1}{2}} t + \eta\right\},$$

so that the motion with a damper has a constantly diminishing amplitude and a slightly altered periodic time. But we saw that in the dynamo the most troublesome oscillations were not the free but the forced ones, and it was pointed out that in default of any external effects the swing became infinite when the natural and impressed periods coincided, or that running became impossible if that condition was approached. With dampers we have quite a different result. Suppose we have a periodic torque on the shaft of the machine given by $D \sin pt$, then the torque equation is

$$K \frac{d^2\alpha}{dt^2} + F \frac{d\alpha}{dt} + L_0\alpha = D \sin pt.$$

Since the "constants" involved are fixed quantities the α must also be harmonic and of the same period as D. Thus the vectors

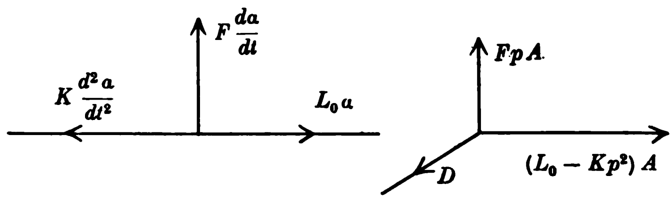


Fig. 62

representing the four torques are as in Fig. 62. Now, with simple harmonic variation, if A is the maximum of α the maximum of $d\alpha/dt$ is pA and of $d^2\alpha/dt^2$ is $-p^2A$. We can therefore draw the vector diagram of Fig. 62 and expressing the fact that D is the resultant of the other vectors we derive

$$D^2 = \{(L_0 - Kp^2)^2 + F^2p^2\} A^2.$$

Making the usual substitutions of q^2 for L_0/K and m for F/K, this gives

$$D = K\{(q^2 - p^2)^2 + m^2p^2\}^{\frac{1}{2}} A.$$

A similar substitution of B for D/Kq^2 for the deviation appropriate to the couple D gives us

$$\frac{A}{B} = \frac{q^2}{\{(q^2 - p^2)^2 + m^2p^2\}^{\frac{1}{2}}}.$$

The result is quite different from the case in which the damping couple is absent. Even with equality of p and q there is still a definite limit to the amplitude A which does not become infinite. It may be noted that the amortisseur also provides another valuable safeguard which will not be clear till we have dealt with induction

motors; if the field magnet by any chance loses its excitation, the amortisseur bars act like the short-circuited armature of such a motor, and the armature currents still being supplied from the mains produce the necessary rotating field, hence it can run on as a motor. This is sometimes used to start up such a machine from rest with reduced pressures supplying the armature; it will run up nearly to full speed, and then lock into step at the end.

39. The Critical Inertia. We have now seen that by suitable means a sudden relative displacement of the dynamo is gradually reduced to zero by damping forces either natural or supplied by an amortisseur, and that it is possible to deal with even forced oscillations, but if the amplitude of the latter gets too large and the approach to the natural period of the machine be too close, a persistent to and fro swing will result. If the damping forces are sufficiently large this may go on without the machine falling out of step, but the impressed couples may be such as eventually to become too large to be controlled by the damping ones; the oscillations will gradually grow in amplitude with corresponding disturbances to the load delivered by the machine, the pressure and other quantities, until the damping couples are overcome and the machine suddenly stops. This period of either persistent "phase swinging" leading possibly to final breakdown in the accommodation is termed "hunting" and must be prevented at all costs. We will now consider how the conditions to avoid this can be investigated, and must therefore return to our equation on p. 89 giving the critical moment of inertia of the flywheel effect, namely

$$K = \frac{VI_sP}{\lambda^2\Omega^3}.$$

This expression can readily be recast in a form more suitable for practical calculations as follows. The moment of inertia is best expressed in kilogram-meter or metric-ton meter units, as given on p. 88. The load-bearing capacity of a machine is usually expressed in terms of the maximum kilo-volt-amperes that it can carry including all the phases in the machine; further, the short-circuit current is most conveniently expressed as being greater than the full load one by some assigned factor say δ, so that $VI_s = \delta$ (V.A.) $= \delta/1000$ (K.V.A.). The performance of any set of machines must be considered at the frequency employed, so that we have $f = PN$, where N is the R.P.S., and finally the speed is

more usually given by N than by Ω. If we denote by K_0 the moment of inertia of the machine in metric tons or 1000 kilograms at 1 meter radius including all the rotating parts such as the fly-wheel, when reckoned per kilo-volt-ampere of its maximum output, the above equation reduces to

$$K_0 = 0{\cdot}004 \frac{\delta f}{\lambda^2 N^4},$$

a very convenient form for practical considerations. If K_0 is required in ton-foot² units the factor is 0·000043. The main uncertainty lies in the value of δ. A reference to Fig. 44 will show that there is considerable difference between the value of the synchronous reactance of a machine on low excitation and on full, so that the short-circuit current on full load excitation may be a good deal larger than on no load. As an example we will consider a range of machines in which the value of δ is 2·5 for no load excitation and 3·5 for full load, all being for 50 periods, and driven by slow speed prime-movers at speeds that may be from 60 to 150 R.P.M. We will also take λ as unity for a reason to be given later on. In Fig. 63 are given two full curves showing the results of plotting the equation for the above two values of δ. The critical value of K_0 for any speed may be taken as being given by any ordinate belonging to the strip between the two curves; if the actual moment of inertia of the plant is such that its K_0 lies above or below the strip bounded by the curves, it escapes

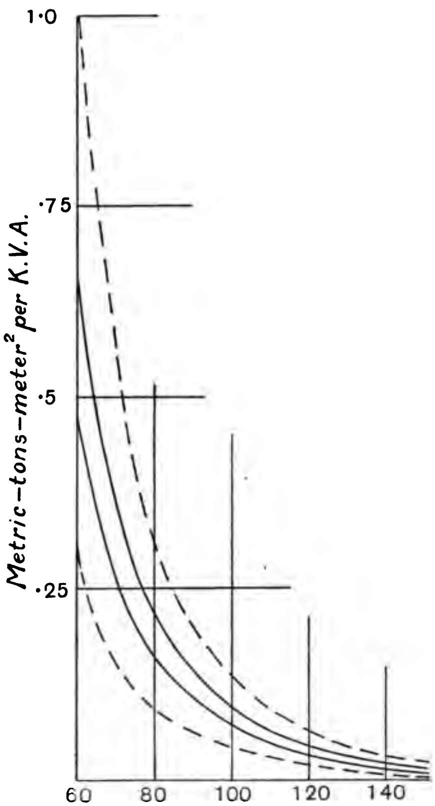


Fig. 63

the critical condition. But we must have a factor of safety, and Dr Rosenberg considers that a safe limit is one corresponding to a ratio of natural to forced periods given by 1·55 for the lower forced periods and hence the reciprocal or 0·65 for the inverse condition. This leads to values of the "reaction quotient" which are respectively 0·7 and 1·75. We then add two further curves shown in dots in the figure, the upper one being such that the ordinates of the top full curve are 0·7 of the ordinates of the top dotted one, and the lower dotted curve such that the ordinates of the full curve are 1·75 times those of the dotted one. The dotted curves then demarcate a broader strip such that we may anticipate danger if the flywheel effect falls inside the broader band thus formed, though practical certainty of hunting is not present until the inner bands are crossed.

We must now briefly consider the prime-mover side of the question. The flywheel of the engine is settled by the "coefficient of fluctuation" permitted to the designer, that is to say, the ratio between the difference between the maximum and minimum speeds in a revolution and the mean speed is assigned. Apart from special conditions the variations of torque occur once per revolution of the engine, and as we are considering only direct drive, this is why the constant λ was taken as unity in the example. The variation of torque is due to many causes, some definite, such as those depending on the geometry of the system, some indefinite, such as possible differences in torque in the two strokes owing to faulty valve setting; in cases where direct-driven pumps are used another cause of varying torque is present. The engine should be so designed and so adjusted as to its valve admissions, etc., that the coefficient of fluctuation does not exceed some assigned fraction say between 1/200 and 1/300 when account is taken of the combined flywheel effect of the whole plant. If calculation from the mechanical data shows that the flywheel provided is such as to lie outside the danger zone shown by the dotted lines, it may be considered satisfactory, but otherwise special precautions may have to be taken such as provision of good amortisseurs. If necessary the flywheel must be increased so as to avoid the zone; this may be secured also by altering the dynamo so as to cause it to have a different value of δ by altering the synchronous reactance. In such way or ways satisfactory running as regards a periodic disturbance for which λ is unity can be secured. It will be seen that if this is done, any

sturbance of higher period will be practically negligible, since λ ppears as a square in the denominator, so that the danger zone r such an harmonic in the disturbing torque is far away from that r the fundamental. But in some cases we have a sort of sub-rmonic; thus a gas engine drive of the ordinary two cycle type volves an impulse every other revolution which may have effect en with multiple cylindered engines; but this means we must t $\lambda = \frac{1}{2}$ or that, other things being equal, we want four times the wheel effect; in fact we shall now have two danger zones to deal th instead of one and the problem becomes much more difficult. we suppose a second set of bands to be added to Fig. 63 having linates four times as large, this would correspond to a prime-ver having a sub-harmonic of half frequency. It will readily be en that the safe area is now much restricted; to get above the w upper curve would mean an enormously large flywheel, and e safe area between the two bands is very narrow. If we can rrow the bands themselves, this area can be increased and such rrowing means a diminution of δ, that is to say a dynamo with ich closer regulation and therefore one that is much more ex-nsive. Such machines should have amortisseur windings.

In rarer cases the sub-harmonic may be even more remote from e fundamental, as for example when turbo-alternators drive ared pumps or are governed by "puff" governors, and in such ses hunting is practically certain.

40. Motor Action. When the question of parallel running an alternator was considered (p. 80) incidental reference was ade to its action as a motor when being brought back into position, d we must now deal more in detail with that motor action. It ll be helpful to refer briefly to the direct current machine. ppose we have a separately excited machine with an armature sistance R working from mains at pressure V. Let it be acting a dynamo with the E.M.F. E; then we have $E = V + RI$. In g. 64 a are shown the mutual relations between the directions of e polarities of the field due to the main winding and that of the mature; the inherent torque is shown by the lower arrow, e direction of rotation Ω; also the state of the polar horns ither weakened or strengthened), the direction of flow of the rrent, and (with the assumed forward brush lead) the direction action of the armature's demagnetising turns by the arrow A.

Leaving everything unaltered, reduce the field or increase V · is larger than E so that we have $V = E + RI$. The direction c armature current will reverse (Fig. 64 *b*) and with it the arm

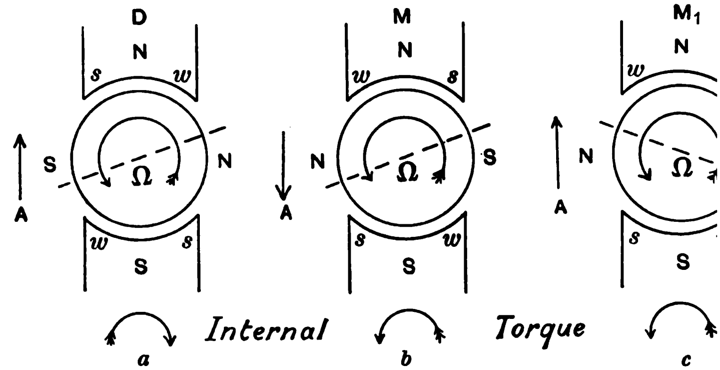


Fig. 64

effects, so that the horns interchange their state, the arrow A p the other way, the inherent torque is in the direction of Ω and l the machine will be delivering power to its prime-mover in: of absorbing it. We know that matters are not left here in a c current machine but that the brush lead is reversed as shown at 64 *c*; this results in everything being left much as before except the direction of A, and that direction only, is reversed, so that armature still demagnetises the this change is made for commut reasons. Now let the machine polyphase alternator, working c practically non-inductive load. have seen that the armature proc a perfectly stationary field in s related to the main field in identi the same way as the correspoı fields are related in the direct cu machine. It will produce the cross-magnetising and weakening o horns, and also a demagnetising (since the current will lag a little

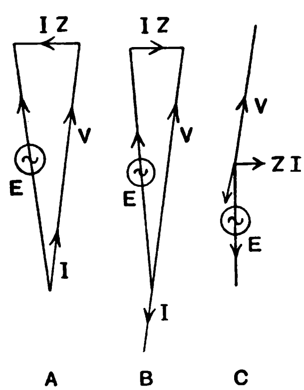


Fig. 65

the nominal E.M.F. owing to the armature impedance. In n all respects Fig. 64 *b* will do again, the space-lead of the bri from the neutral line in the direct current case corresponding t

time-lag of the current in the alternator. But the relation between the E.M.F., P.D. and drop will now be vectorial; we will take the drop as being equal to the product of the current and the synchronous impedance, and if the latter be denoted by Z, the drop will be IZ. Suppose now that the prime-mover is suddenly shut down, the current will fall to zero, will rise in the opposite direction and thereby establish a new stationary field the same as the original one but opposite in direction as shown at Fig. 64*b*. Just like the direct current machine the alternator becomes a motor, the polar horns are similarly affected, the torque is in the direction of Ω, the current enhances the field flux owing to the reversal of the current in the demagnetising belt; if the current were more lagging than on the assumed non-inductive external load, this latter effect would be larger. On the other hand with a leading current which magnetises the poles in the dynamo, the reversal of the current band in the motor causes a leading current to demagnetise the poles. There is nothing corresponding to Fig. 64*c*, where the brushes of the motor were purposely put back to produce a demagnetising effect, as there is no need to provide for a reversing field for the commutation of the armature currents. Our vector diagram must now express the fact that E is no longer the sum of V and the drop, Fig. 65 (A), but that V is the sum of E and the drop, as shown at Fig. 65(B). But the reversal of the drop vector entails that of the current vector and we must briefly consider various ways of drawing the figure. The various forms depend merely on whether one prefers to depict the quantities in terms of resultants or equilibrants. In the dynamo diagram, Fig. 65(A), we depict IZ as the result of subtracting V from E, while in the motor one (B) we subtract E from V. If we wish to emphasize the fact that the E.M.F. is nearly opposite in time-phase to the P.D. we can use Fig. 65(C) where the E.M.F. vector is reversed in direction. This has the advantage of depicting more accurately the current and pressure relationships of the motor, and gives, as is otherwise evident, the same phase difference as the dynamo, Fig. 65(A). The most convenient is to use A for both conditions in which form the motor E.M.F. is looked on as that portion of the P.D. that is demanded by the motor.

A difficulty is felt by students in reconciling the fact that a leading current demagnetises the motor's field while a lagging one magnetises it with the apparent neglect of these points in the treatment. In nearly all considerations of synchronous motors,

however, we use the concept of the synchronous reactance, and it must be remembered that that constant includes in itself all the armature effects as well as the true reactance. Thus on referring to Fig. 46 as representing the dynamo operation we attain to the required E.M.F. by passing successively through the various pressure vectors OV, VR, RX, the latter being that for the synchronous reactance, X, which includes in itself the cross effect and the demagnetising or magnetising one. But with the motor we start with OE and successively subtract XI and RI arriving the opposite way to the E.M.F. of the motor now given by OV. Thus the magnetic effects are all reversed automatically, but we can still approximately represent them by the synchronous reactance. Hence when this quantity is used, we implicitly assume that we have taken into account *all* the armature effects, and that the E.M.F.'s we are dealing with are the actual applied pressure and the nominal E.M.F. of the motor due to its fixed excitation as derived from its saturation curve. Care must be taken to keep this point clearly in mind.

41. Some Motor Problems. Many problems of interest and importance can be solved directly by the triangle of pressures, and a few examples will now be considered: in all we may take the length C of the vector IZ as a measure of the current, while the direction of the current will make the angle α of internal lag with IZ, that is to say, in phase with the resistance-vector. It must always

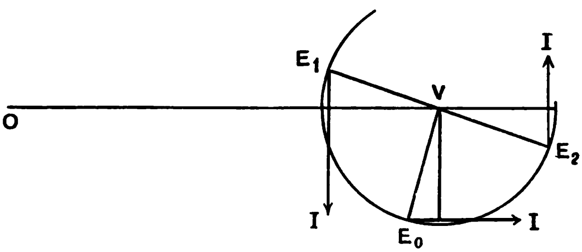


Fig. 66

be remembered that there are definite limits to the possible length of C and to the E.M.F. of the motor, the former being limited by the maximum current the armature can carry, the latter by the maximum excitation the fields can carry, and thus the diagrams cannot be taken as giving under all conditions the actual possible values of C and E that can be obtained in a real machine. Let us now enquire what are the possible limits of the motor E.M.F. for a given pressure

and current: this implies that the length of C is fixed, so that we can draw the circle shown in Fig. 66. The possible limits of the phase angle between the current and the P.D. lie between a lag of 90° and the same lead, so that the limits for E are as shown, OE_1 being the E.M.F. corresponding to a lag and OE_2 to a lead of 90°; the condition for inphase current is given by the point E_0. Between E_1 and E_0 the current lags on the P.D.: beyond E_0 and up to E_2 it leads, though E_2 might well be an unattainable value if the current were large. It must follow that for any lag angle between E_1 and E_0 there is a corresponding lead in the other section.

Another question might be:—how must the motor's E.M.F. vary if the load has a constant phase angle? In Fig. 67 take OV to be

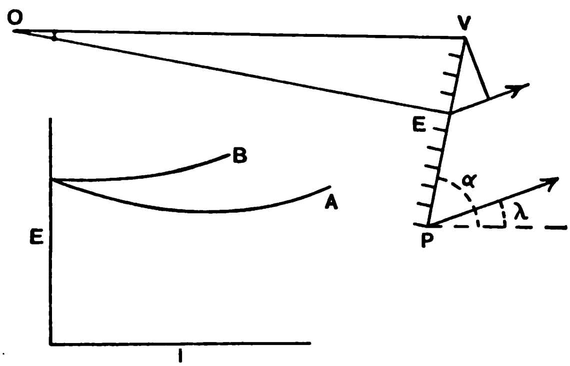


Fig. 67

the constant applied P.D. and draw a line VP at the angle $(\alpha - \lambda)$ to it. Then different currents up to the maximum allowable will be represented by points on VP, so that for one of them, E, OE is the required E.M.F. It will be seen that if we plot this E.M.F. against the current we get a bowed curve as shown below at A, in fact this is a total characteristic for the motor at that phase angle. Similar curves can be drawn for any angle such as OVE. Directly this angle becomes a right angle, the bow disappears as shown at B, and the E.M.F. must therefore be made to rise with the current in order to keep λ constant with constant V. In a machine with a large value of α this means that when the load is non-inductive, to keep it in that state we must adjust E to rise continuously with the current; this will be seen to be of practical importance later on, for it shows that if a machine has too small a reactance for the required condition of the E.M.F. rising with the

current on inphase load to be naturally fulfilled, we can greatly improve the machine by adding reactance in some way to its circuit.

In the above no stipulation was made as to the power that is being supplied or delivered; we will take two such cases. Firstly supposing that the motor has fixed excitation so that E is constant, and is taking constant power from the mains, how must their pressure vary? In Fig. 68 let OE be the E.M.F., and let the inphase current that will account for the assigned power be OI. Draw EV at the angle α to OE and of the length to give the drop C; the P.D. required on inphase current is then OV. If the perpendicular II_1 be drawn as shown, the same power would be absorbed with the current OI_1 lagging by the angle λ or by the equal current OI_2

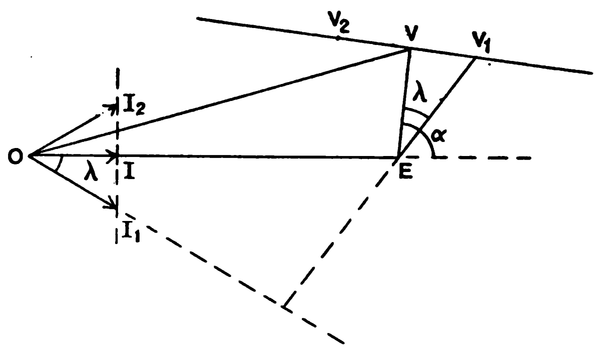


Fig. 68

leading by the same angle. To find the P.D. required for the first, draw a line through V perpendicular to EV, and set off EV_1 at the angle λ to EV. We then have $EV_1/EV = I_1/I$, or EV_1 is the new drop in the armature; hence OV_1 is the required P.D. But since the triangles OII_1 and EVV_1 are always similar, as I moves along its straight line up to the maximum allowable value, so V moves along the line V_1VV_2, and the latter can be taken as a line of constant input into the motor. Similarly the point V_2 corresponds to a leading current; all points between V and V_1 indicate a lag, the others a lead.

The next example is a more practical case. A motor is running on mains kept at a constant pressure V and is supplying a fixed amount of power to the shaft at an assigned current; what must be the E.M.F. to which it is excited? The load per phase must be taken to include the actual nett shaft load P_u doing useful work

and the waste load P_L required for friction, etc. The machine being synchronous, the mechanical part of P_L is naturally very nearly constant, and we will assume that the part required for hysteresis and eddies is also constant, that is to say, we assume that the alteration in the field of the motor either due to its own reactions or to outside change in excitation is not excessive. Let $P = P_u + P_L$, then P is equivalent to a pressure V_0 always in phase with the fixed current I. Draw OI to give the direction of the given current and OQ at the angle α to it, that angle being the internal lag angle or one whose tangent is the ratio of the synchronous reactance to

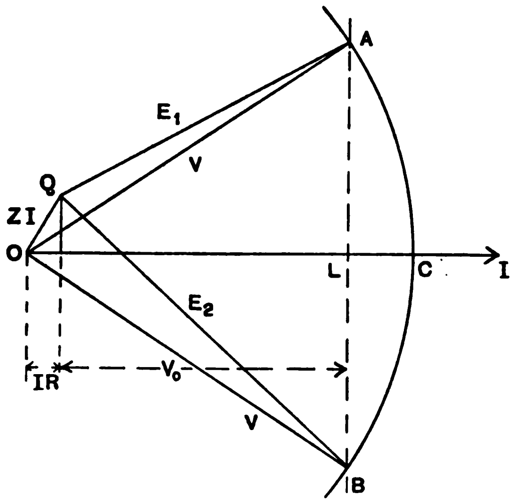


Fig. 69

the ohmic resistance of the armature. Then the drop will be along OQ and of the fixed amount ZI, where Z is the synchronous impedance. Select a scale of pressure and set off OQ equal to this constant ZI, and the distance OL equal to $(RI + V_0)$ so that OL is the constant inphase pressure. Draw a perpendicular to the current line through L and a circle of radius V with O as centre, this will in general cut the perpendicular in A and B. Join these points to O and Q, and it will be seen that two solutions result, giving two possible E.M.F.'s. One of these with the higher value, E_2, involves the current leading on the pressure by a certain angle λ, the other or lower one E_1 involves the same lag. Hence the given load can be taken by the motor either with over-excitation and a

leading current or with under-excitation and a lagging current. If a set of loads are assumed, we get a similar construction for each, the no-load condition corresponding to a line through Q leaving only the ohmic loss to be supplied, and hence in modern machines this line nearly passes through O, and the two E.M.F.'s differ from V merely by the internal drop taken algebraically. The maximum load corresponds to a line tangential at C and leads to the two triangles shutting up into one with no phase difference as regards V and a small lead of current as regards E. It will readily be seen that if we enquired what will be the curve connecting the E.M.F. to which the motor is excited and the line current for a given power, the same diagram could be used, only for each current we must set off the proper value of OQ and that of OL; thus with the same power as that corresponding to the figure if we suppose we are to use 4/5ths of the current, we must diminish OQ in that ratio and increase V_0 in the same ratio. If this is carried out it will be seen that for any assigned power and varying currents there will be two possible E.M.F.'s for each current so that the relation between the line current and the E.M.F. of the motor is a "V" shaped curve as shown in Fig. 73; there will be one such curve for each load. Further, for each load there is a minimum line current, and from the method of derivation this corresponds to the case of equality of the E.M.F.'s and also corresponds to the current being inphase with the P.D. Hence all points between the dotted line of C and the axis of Fig. 73 will correspond to lagging currents, all the others to leading ones. The property of an over-excited synchronous motor taking a leading current is of much importance, as it provides a means of improving the power factor of a system, indeed such a machine running light is used to act as an adjustable condenser for that purpose alone.

42. The Power Circles and "V" Curves. It will be of interest to go more fully into this question, and although it will be seen that the results have a comparatively limited range of practical application, they help to clear up the details. We will therefore consider more closely the case of a motor excited to an E.M.F. E and running on mains at constant P.D. V with various loads and currents. As there is no gain in simplicity by using the reactance only, we will take the armature's impedance Z into consideration instead of the synchronous reactance only, and will

specify the armature's properties by Z and the internal phase angle α, the latter being in practice nearly 90°. Further we will denote the difference of phase between E and V by θ. A possible relation between the three pressures V, E and ZI for a current I will then be as in Fig. 70. Let β be the angle between E and I as shown; produce BC and draw a line FD making the angle β

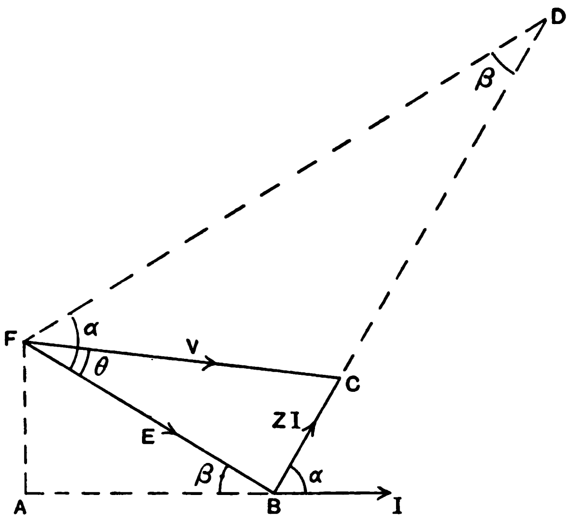


Fig. 70

with BC. Since the angles at B and those of the triangle FDB are both two right angles, it follows that DFB is α. Project the sides of the triangle FBC on to FD and we have

$$E \cos \alpha + ZI \cos \beta = V \cos (\alpha - \theta).$$

But the gross power that the motor is delivering, being the product of the current into the inphase pressure of the motor, is $E \cos \beta \times I$, so that on eliminating β we have

$$P = \frac{1}{Z}\{VE \cos (\alpha - \theta) - E^2 \cos \alpha\}.$$

If desired a similar expression can be deduced for the input, but we do not require it.

It is useful to have the current measured on the same scale as the pressures; but the line BC gives the drop so that ZI is a measure of I and when we wish to specify it as a voltage we will as before denote it by C, the current being then given by $I = C/Z$.

Supposing the motor is working steadily with nearly constant assigned excitation and power, and that there is a very small alteration in the latter, either an increase or a decrease, can the motor respond and supply the demand, in other words, what is the condition for stability of the given state? The only way in which the system can change is by a small accommodation in the angle θ, so that we will differentiate with regard to that angle, resulting in

$$\frac{dP}{d\theta} = \frac{EV}{Z} \sin (\alpha - \theta).$$

Now this is positive up to $\alpha = \theta$, so that up to that limit of the angle between the P.D. and the motor's E.M.F. the motion is stable for all possible powers up to the maximum that can be delivered. In an ideal case the latter is given by the maximum possible value of the former expression for P, namely when $\cos (\alpha - \theta)$ is unity, involving a motor E.M.F. of the value $E = V \sec \alpha$ which is enormously greater than any possible actual value since α is nearly 90°; hence we can state that for all possible loads that can be expected the action is stable.

It may be noted that dP in the above is the synchronising power dealt with formerly (p. 85). For if we remember that α was there taken as 90°, θ as being small, and σ as the change in θ, while E and V were the same, this leads to $dP = \sigma V^2/X$, or our old expression, if reactance be substituted for impedance.

The condition that must be fulfilled in order that the synchronising power may be a maximum can be found as follows. The expression just found can be written in the form

$$dP = \frac{V}{Z} \{(E \cos \theta) \sin \alpha - (E \sin \theta) \cos \alpha\} \, d\theta.$$

But the maximum value of this power for the given machine will be that for which the negative term vanishes; since θ is necessarily positive this leads to the expression $dP = \frac{VE\,d\theta}{Z} - E \sin \alpha$ giving the maximum synchronising power for the machine, or since $\sin \alpha = X/Z$ it is $X/Z^2\, VE\,d\theta$. What we may call "the maximum-maximum" possible for all machines having the same reactance is then given by the condition that must be fulfilled in order that X/Z^2 may be itself a maximum, that is to say, $X = R$. This reactance is far too small for a modern machine, but the relation is one that might well be utilized when it is necessary to arrange that different

stations have to be worked in parallel, as it is not very difficult to adjust the constants of the inter-connecting cables.

We can now readily derive a convenient graphical representation of the power of the motor, which incidentally leads to a more complete determination of the "V" curves, as follows. Set off to scale the triangle OGM, Fig. 71, connecting the fixed P.D., and any E and IZ corresponding to a definite power. Draw two lines through the ends of OG to meet in C such that the basal angles of the triangle are each the internal phase angle α. Draw a perpendicular from M on to OC and let p and q be the lengths OL and LM. Then we have

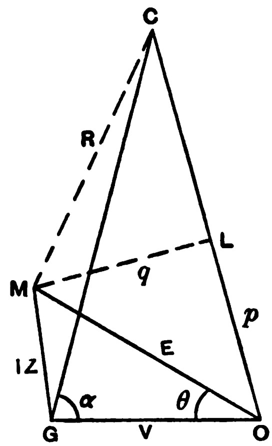


Fig. 71

$$p^2 + q^2 = E^2 \text{ and } p = E \cos(\alpha - \theta),$$

but the power is given by

$$P = 1/Z\ \{VE \cos(\alpha - \theta) - E^2 \cos \alpha\}.$$

Hence it follows on substitution that

$$ZP = Vp - (p^2 + q^2) \cos \alpha;$$

hence $$p^2 - Vp \sec \alpha + q^2 = - ZP \sec \alpha,$$

or $$(p - V/2 \sec \alpha)^2 + q^2 = \sec^2 \alpha\,(V^2/4 - PZ \cos \alpha).$$

But if r is the resistance of the armature we have $Z \cos \alpha = r$, so that the right-hand side becomes $\sec^2 \alpha\,(V^2/4 - Pr)$. But we have $OC = V/2 \sec \alpha$, and LC is $(V/2 \sec \alpha - p)$ while q is LM. Hence if we denote CM by R, we get $R^2 = LM^2 + LC^2$,

and hence $$R = \sec \alpha\,(V^2/4 - Pr)^{\frac{1}{2}}.$$

This shows that for a given power the locus of M is a circle, hence the different loads taken by the motor will be represented by a set of such circles with C as centre.

Certain interesting points are now made clear; thus, when M is inside the triangle OGC we have the current lagging on the P.D., when it reaches the line OC, θ is equal to α so that the line OC is the limit of stability; also, the rest of the circle outside the triangle corresponds to leading currents. This gives a clear idea of the general nature of the problem, but unfortunately numerical results are based on a knowledge of the armature resistance r, which is a

rather problematic and essentially variable factor, hence no really reliable construction is possible.

But since the construction gives a very clear outline of the operation of the motor, we will apply it to the small machine whose constants were given on pp. 56 and 62. The two lines OC and QB, Fig. 72, were drawn at the angle α to the line OQ which is to scale the applied pressure of 200 volts. The radii corresponding to various percentages of full load were then drawn with the meeting point of these lines as centre, the loads taken being nett outputs, with a small percentage deducted for the constant mechanical losses. The current being expressed in terms of the corresponding drop from the relation $C = ZI$, another set of circles are drawn with Q as

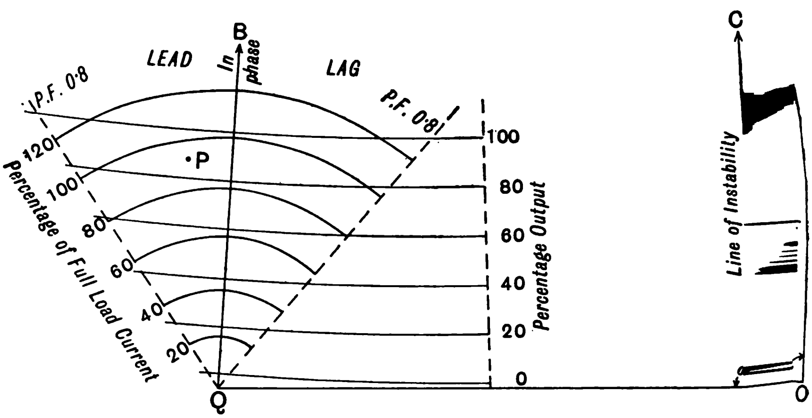
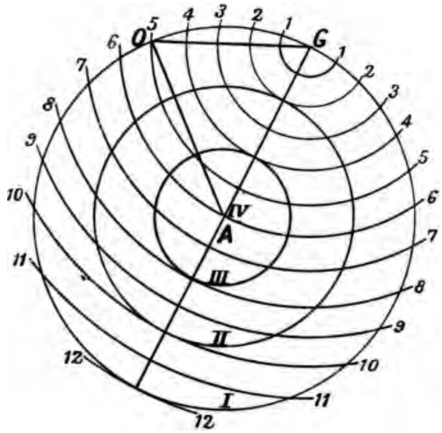


Fig. 72

centre such that they give the current as a percentage of its full load value. Since most practical conditions of operation lie between certain limits of power factor, namely above about 0·8 for lag or lead, two radii are drawn making angles with the inphase line QB such that their cosines have that value. The whole operation can now be followed very conveniently; within such limits as will not permit a current more than 20 per cent. in excess of the normal full load one to be used, and with the given limitation in the power factor, we are kept inside the sector bounded by the two radii and the final current arc. We can take any point such as P inside that area and determine approximately the complete conditions of operation, especially if a full series of power factor lines are drawn. The relation between any two of the variables, the

others being fixed, can be followed with ease. *PQ* will be the current, *OP* the necessary motor E.M.F., the large circle through *P*



***OG*$=$*V*, *AOG*$=a$, current or drop circles 1 to 12, power circles *I* to *IV*.**

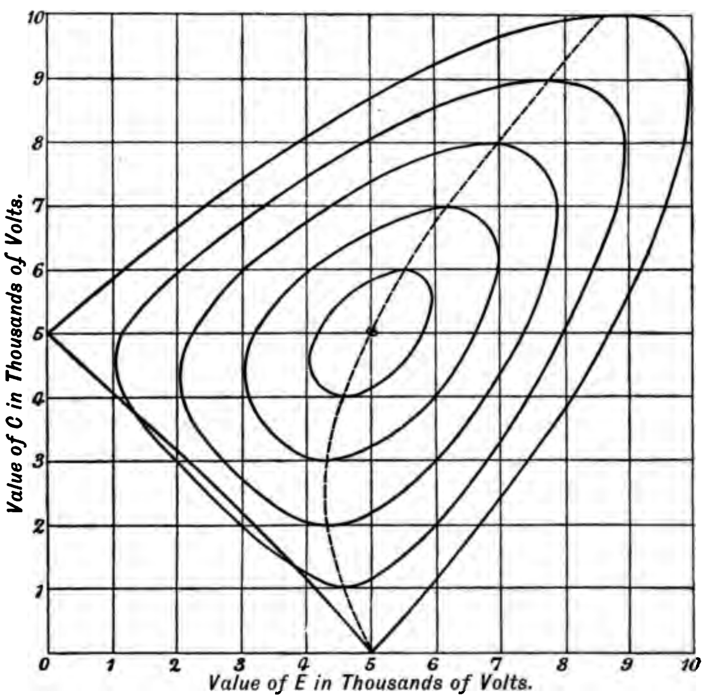


Fig. 73

gives the percentage of power delivered, about 90 per cent., and so on. It will be seen that the condition of instability is far outside the useful part of the diagram.

Although it follows that the useful part of the complete diagram of Fig. 71 is a very small fraction of the possible whole, it is of interest to give the complete curves connecting the motor E.M.F. as measured by *OM* in that figure and the corresponding current as measured by *GM*. The full set of power and current circles are given in Fig. 73 for a purely hypothetical case where the P.D. was taken as 5000 volts and the internal phase angle as 60°. The "*V*" curves are easily picked out by pricking off along the axes the corresponding values of *E* and *C*, that is to say, those lengths at the points where any two of the respective circles cut, one in each set. The resulting curves are quartics; the parts concave to the axis of *X* correspond to a stable action, the others to an ideal unstable action on the wrong side of *OC*, Fig. 71. Two points may be mentioned: the zero power lines are parts of two ellipses as shown in Fig. 73, which can quite simply be shown from the relation between the sides of the E.M.F. triangle when the condition of no output is substituted. Again, the maxima and minima of the "*V*'s" lie on the dotted line which is itself a quartic.

43. Hunting of Motor. We will now consider the effect of a disturbance in the driving couple of a synchronous motor.

On p. 85 we considered the effect of a slight displacement of a dynamo, relative to its steady position, both when the displacement was a definite one which was damped out by frictional or other dissipating couples, and when it was a forced one impressed by the prime-mover. Similar considerations apply to the motor, except that the impressed variations in couple are not due to the imperfections in torque of the prime-mover but to imperfections in the constancy of the periods of the supply pressure. This introduces an important difference in the problem.

Let *OS*, Fig. 74, be a vector rotating with absolute constancy at a rate equal to the ideal constant applied periods. At any moment in a swing the armature will make an angle α with that vector, as in the former problem. But here the applied pressure may have swung away from *OS* owing to variations in its frequency, and hence will be at some angle like ζ to the proper position. Now the torque demanded by the moment of inertia depends solely on

he angle α, the synchronising torque and the damper torque, iowever, depend on the angle $(\alpha - \zeta)$ and its rate of change, so hat the torque equation is now

$$K\frac{d^2\alpha}{dt^2} + F\frac{d}{dt}(\alpha - \zeta) + L_0(\alpha - \zeta) = 0.$$

'he implied condition is that ζ varies in a simple harmonic manner ɔ that $\zeta = D\sin pt$. But as before this involves α being simple

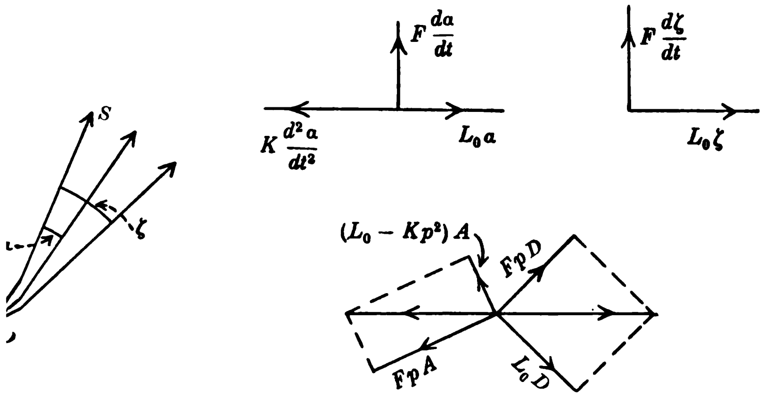


Fig. 74

ɐrmonic also, with the same periodic time corresponding to p, ɔd a definite maximum, A. Separating the two angles we have

$$K\frac{d^2\alpha}{dt^2} + F\frac{d\alpha}{dt} + L_0\alpha = F\frac{d\zeta}{dt} + L_0\zeta,$$

› that Fig. 74 (top) gives the direction of the quantities. But as in ıe example on p. 91 we can substitute $-Ap^2K$ for the maximum ˸ $K\frac{d^2\alpha}{dt^2}$ and AFp and DFp for those of $F\frac{d\alpha}{dt}$ and $F\frac{d\zeta}{dt}$. Further, ıe resultant action of all the couples must be zero, so that we can ʇuate the separate maxima involved in the two angles. This ·ads to

$$\{(L_0 - Kp^2)^2 + F^2p^2\}A^2 = (L_0{}^2 + F^2p^2)ID^2.$$

[aking use of our customary substitutions, namely $q^2 = L_0/K$ and ı $= F/K$, this leads to

$$\frac{A}{D} = \sqrt{\frac{q^4 + m^2p^2}{(q^2 - p^2)^2 + m^2p^2}}.$$

This result differs radically from that for the dynamo with varying prime-mover couples, inasmuch as the quantity m appears in the numerator, so that it is possible for A to be larger than D. The maximum value occurs when $p = q$ and is then

$$\sqrt{1 + \frac{q^2}{m^2}} \text{ or } \sqrt{1 + \left(\frac{Kq}{F}\right)^2}.$$

This increase in the displacement angle of the machine over that of the disturbing factor may be quite considerable.

44. The Rotary Converter. One of the most important forms of synchronous machine is the Rotary Converter which consists of a synchronous motor and a direct current dynamo operating through a common armature. It consists of the inversion of the machine mentioned on p. 8, where alternating currents were delivered from slip rings attached to suitable equidistant points round the armature, from which the appropriately phased alternating currents could be taken when the machine was driven as a direct current motor. When used for the opposite purpose it must first be brought up to speed like any other synchronous machine, and can then be put on the mains, but usually the pressure required from the direct current side (which is necessarily definitely related to the slip ring pressure) is much smaller than that of the alternating supply, thus entailing the use of intermediary transformers. When running, the machine will excite in the ordinary way, and direct current can then be taken from the commutator. It must be noticed that the speed is here quite fixed, and hence no speed-voltage regulation is possible; further, any alteration of the pressure on the direct current side entails a proportional alteration on the alternating current one; but since the alternating supply is also at nearly constant pressure as well as at constant periods, this means that either automatic adjustment of phase must be possible between the E.M.F. of the slip rings and the P.D., or else that suitable devices must be resorted to in order to make up the difference of pressure. The supply of a three-phase rotary is quite simple, the armature forming a mesh which is necessarily balanced as to load; it can be supplied either by three separate transformers with secondaries meshed on the rings or by a single three-phase transformer. But it is quite easy to make the supply six phase, all that is necessary is to provide three other transformers with their second-

ary mesh exactly opposite in phase to that of the first set and to connect the first to the points 1, 3, 5 in Fig. 75, and the second to 2, 4, 6. But a far simpler method is to have only three secondaries, the primaries being, say, in star, and place the three secondaries across diagonal points as 1 and 4, 2 and 5, 3 and 6 which gives a precisely similar effect. It is readily seen that four-phase rotaries can be fed from two phases at 90° provided the secondaries of the transformers are placed across the corresponding diameters of the armature.

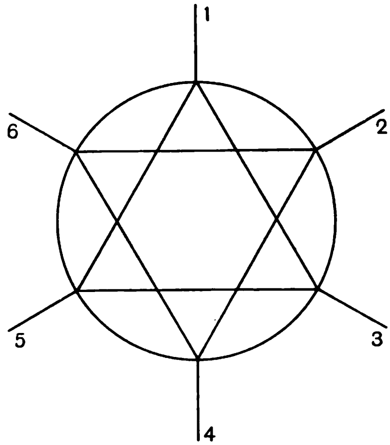


Fig. 75

With a sinoidal field form curve it is quite easy to find the relation between the direct current pressure E_0 and that between any pair of rings, say between adjacent ones. For if there be m tappings or rings, the angle of polar pitch being as usual π, the angle between the tappings will be $2\pi/m$. Let the maximum E.M.F. in a wire of the armature be e_0 and the wires per radian σ, the field form being sinoidal entails that the mean E.M.F. per wire is $\frac{2}{\pi} e_0$, and hence the direct current pressure between A and B is $\frac{2}{\pi} e_0 \times \sigma\pi$ or $E_0 = 2\sigma e_0$.

Now let the wires in a phase be at such a position that the alternating E.M.F. is a maximum. Reckoning the E.M.F.'s from the centre of the pole we see that the maximum E.M.F. in the coil with a sinoidal field form is

$$\sigma e_0 \int_{-\pi/m}^{\pi/m} \cos \xi \, d\xi \quad \text{or} \quad 2\sigma e_0 \sin \pi/m,$$

that is $E_0 \sin \pi/m$; hence the virtual E.M.F. is given by

$$E = E_0/\sqrt{2} \sin \pi/m,$$

so that the ratio of AC to DC volts is as given below.

Three phase	0·612
Four phase	0·5
Six phase 1 to 2	0·353
,, ,, 1 to 3	0·612
,, ,, 1 to 4	0·706

With actual field form curves the ratio is somewhat different; thus with normally shaped poles in a three-phase machine in which the ratio of pole breadth to pole pitch is 0·8 the E.M.F. ratio is 0·6, with a pole ratio of 0·7 it is 0·62 and with one of 0·6 it is 0·65, and similarly with the others given in the table.

The current flowing in each section of the armature can be easily found; for let the direct current taken be I_0 and neglect ohmic drops, the power must be equally divided between the m phases, and if I is the virtual current in each and E the pressure between rings we must have $E_0 I_0 = mEI$, or since

$$E = \frac{E_0}{\sqrt{2}} \sin \frac{\pi}{m}, \quad \text{it follows that} \quad I = \frac{\sqrt{2}}{m} \operatorname{cosec} \frac{\pi}{m} I_0.$$

The current flowing into the rings can be found from the ordinary three-phase relations, thus with the three phase it will be $\sqrt{3}$ times the armature current, that is $0{\cdot}94 I_0$, with the four phase $\sqrt{2}$ times, that is $0{\cdot}71 I_0$, and with the six phase half that with the three or $0{\cdot}47 I_0$. These ratios must in practice be increased with normal poles corresponding to the alteration in E.M.F. ratios, and also to allow for the loss of power in driving the armature round on light load against the rotational losses.

45. Heating of a Rotary. A very important matter is a comparison of the load the machine can carry when used as a rotary with its full load as an ordinary direct current dynamo. The former may be considerably the larger since the alternating currents on the whole are flowing in the armature wires in the opposite direction to the direct current. The question becomes then one of comparing the ohmic losses in the two conditions. For simplicity we will neglect for the moment the small current necessary to drive the machine light, in other words, we will assume the direct and alternating powers to be the same, so that with I in phase with E the relation $I = \frac{\sqrt{2}}{m} \operatorname{cosec} \frac{\pi}{m} I_0$ holds good. But we can easily allow for any phase angle: for if such an angle λ be present, to get the same power we must increase the current in the ratio $\sec \lambda$, so that the virtual alternating current in the armature is $\frac{\sqrt{2} . I_0}{m} \operatorname{cosec} \frac{\pi}{m} \sec \lambda$, or its maximum value is $\frac{2 I_0}{m} \operatorname{cosec} \frac{\pi}{m} \sec \lambda$. But this current varies harmonically with the time; let the position of the centre line of any phase relative to the centre line of the pole, the position of maximum E.M.F., be given by θ (Fig. 76); then with unity power factor the maximum current

would occur when θ is zero and the current at any other time would be given by $I_m \cos \theta$. But there is a lag of current after the E.M.F. and hence the real expression for the current at the instant corresponding to the position θ is $I_m \cos(\theta - \lambda)$. Now with a two circuit winding half the direct current flowing in at a brush will pass down any wire, the actual current in the phase at the instant given by θ is thus

$$i = \left\{\frac{I_0}{2} - I_m \cos(\theta - \lambda)\right\} = \frac{I_0}{2}\left\{1 - \frac{4}{m} \operatorname{cosec} \frac{\pi}{m} \sec \lambda \cos(\theta - \lambda)\right\}.$$

This is true for every wire in the phase; let any wire be specified relative to the centre of the phase by the angle ϕ as shown.

Then, to find the mean rate of heating in the given wire we must first determine the mean square of the current as the coil turns through half a period, that is through π. This is an important matter as it enables us to trace out the distribution of heating round the phase winding. We can then again determine the mean of the last value for the whole phase, and this will give the average heat produced in the whole armature.

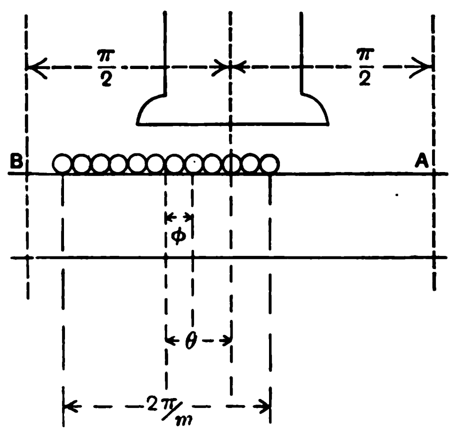


Fig. 76

To find the first, the heat generated in the wire at the angle ϕ relative to the centre of the phase, write for shortness $\beta = \frac{4}{m} \operatorname{cosec} \frac{\pi}{m} \sec \lambda$. The average heat generated in the given wire will be proportional to the mean of the square of the current taken over half a period. It follows that since the time is proportional to the angle θ this mean will be given by integrating the expression for the square of the current from the position when the given wire is at A till it is at B, that is between the limits of θ given by $\pi/2 - \phi$ and $-(\pi/2 + \phi)$, and then dividing by π so that the heat generated in the wire defined by ϕ will be given by

$$\frac{I_0^2}{4} \frac{1}{\pi} \int_{-(\pi/2+\phi)}^{\pi/2-\phi} \{1 - \beta \cos(\theta - \lambda)\}^2 \, d\theta.$$

Since the indefinite integral of the bracket is

$$\theta - 2\beta \sin(\theta - \lambda) + \beta^2 \left\{\frac{\theta}{2} + \frac{\sin 2(\theta - \lambda)}{4}\right\},$$

and the heat generated with the steady current only would have been proportional to $I_0^2/4$, it follows that the heat produced in the given wire when rotary converter action takes place bears to that produced when acting as a dynamo the ratio

$$1 - \frac{4\beta}{\pi}\cos(\phi + \lambda) + \frac{\beta^2}{2}$$

or $$1 + \frac{8}{m^2}\operatorname{cosec}^2\frac{\pi}{m}\sec^2\lambda - \frac{16}{\pi m}\operatorname{cosec}\frac{\pi}{m}\sec\lambda\cos(\phi + \lambda).$$

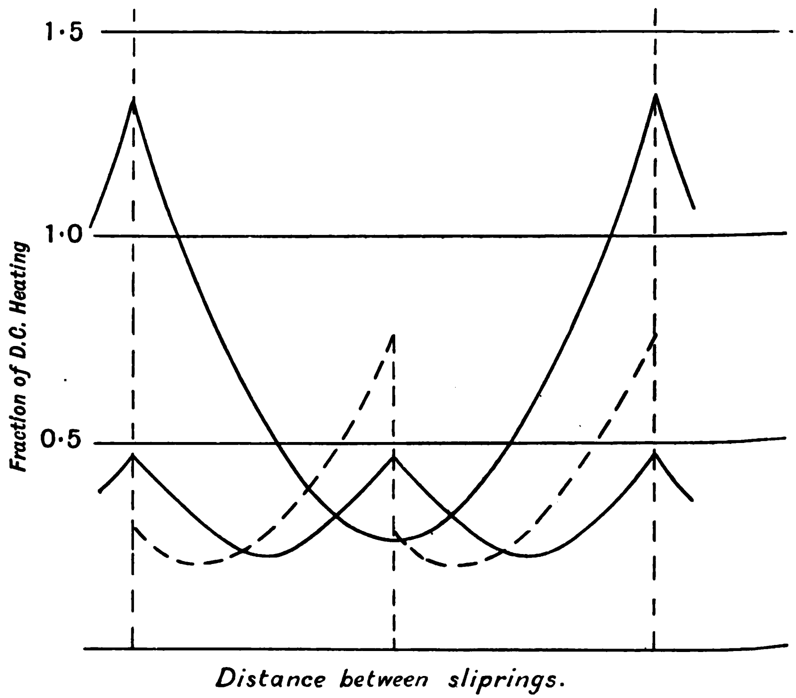


Fig. 77

In Fig. 77 are given curves showing the value of this ratio for a three-phase machine when the power factor is unity, and for a six-phase machine for power factors unity and less than unity. It will be seen that on the whole the heat produced is less: also, the wires near the tappings get the hottest.

To compare the total heat generated in a phase with that produced when acting as a dynamo we must take the mean of the last expression over the distance occupied by the phase, namely $2\pi/m$, and between the limits subtended by the phase, namely from π/m to $-\pi/m$. The only part affected by the integration is the harmonic part and since we have

$$\frac{m}{2\pi}\int_{-\pi/m}^{\pi/m} \cos(\phi + \lambda)\, d\phi = \frac{m}{\pi}\sin\frac{\pi}{m}\cos\lambda$$

it follows that the required ratio is

$$1 - \frac{16}{\pi^2} + \frac{8}{m^2}\operatorname{cosec}^2\frac{\pi}{m}\sec^2\lambda.$$

The reciprocal of this expression may be taken as expressing the proportionately greater load that can be carried by the machine under ideal conditions of equality of power, and with unity power factor it leads to the fact that a three-phase machine can carry 1·32 times the current for the same heat as a dynamo, a four-phase one 1·62 times, and a six-phase one 1·92 times.

The fact that a small current is required to drive the machine light can be combined with the expression for the power factor in a convenient modification of the formula. For in the case considered we have $\sec^2\lambda = 1 + \tan^2\lambda$. Now $\tan\lambda$ is the ratio of the wattless component to the power component of the current in the first case, and is of course known from the lag angle. If we write η for the efficiency ratio of the armature from the A.C. side to the D.C., a quantity known very approximately for any machine, it follows that the power component must be multiplied by $1/\eta$ in order to allow for the power to drive the armature light, and hence if we write $U^2 = 1/\eta^2 + \tan^2\lambda$, the ratio can be expressed in the form

$$1 - \frac{16}{\pi^2} + \frac{8U^2}{m^2}\operatorname{cosec}^2\frac{\pi}{m}.$$

It will be seen that the possible load diminishes rapidly as the angle of phase increases. Thus it can readily be shown, by substituting 3 for m in the above ratio, that a three-phase rotary has exactly the same heat production as the corresponding dynamo when the power factor is 0·8. The great increase in capacity resulting from merely increasing the number of rings is also to be noted.

Like any other synchronous motor the armature currents in

the various phases produce a stationary belt of M.M.F.; when the current taken is at unity power factor, this belt is entirely cross-magnetising, but the direct currents produce an exactly opposite cross-magnetising effect since they are dynamo currents; it follows that the nett cross-magnetising effect is practically zero, and hence the field distortion with its consequent commutation difficulties is much reduced.

46. Regulation. In default of the use of external devices the ratio between the alternating ring pressure, or, what is the same thing, the supply pressure, and the direct current E.M.F. is practically a geometric constant. But it is usually necessary to provide that the direct current pressure may be adjustable to a considerable extent, as in tramway systems. This involves a corresponding alteration in the ring pressure which can only be provided for by means outside the rotary. We can cause the direct current pressure to rise automatically on load by compounding the machine in the ordinary manner, but we must with that provide a means by which the ring pressure can equally rise while the alternating supply pressure remains constant. There are several methods of procedure; if we fix to the rotary shaft a small alternator provided with an armature of the same number of phases as the rotary and connect its several armatures in series with the rings, either between the tappings and the rings themselves or between the transformer's secondaries and the same, this machine when suitably excited can supply the necessary extra pressure. We can either use it alone and utilize the natural transformation ratios of the rotary to give the required increase in direct current pressure, or make it auxiliary to a series winding. This auxiliary machine is called a synchronous booster; it may be either separately excited, in which case by reversing the direction of excitation the boost can be either up or down, or it can be made automatic by exciting it in series from the mains delivering the direct current. Another method is to take advantage of the facts proved on p. 99, where we saw that it was possible to get a load of varying amount at constant P.D. and definite phase angle to be taken by a synchronous motor with a rising E.M.F. of the motor if it had sufficient reactance in circuit. This reactance may be supplied by means of special reactance coils in either circuit of the supply transformers, but is most simply provided by causing those transformers to have excessive leakage

so that their normal regulation is bad. This is secured by non-interleaving of the windings or even by the insertion of small "leakage packets" of iron so placed in the magnetic circuit as to increase suitably the natural stray fields.

A third method is to utilize the fact that the ratio between the mean E.M.F. of any dynamo and its virtual E.M.F. depends essentially on the field form curve; if the shape of this is adjustable, a very considerable range of the ratio between the two can be secured, the direct current E.M.F. depending on the mean value of the induction while the alternating one depends on the mean square. Suppose, then, we split a single pole into three, and let each part be capable of distinct excitation independently of the others. If the three be equally excited the field form will be practically of the ordinary flat type; now weaken the middle one and strengthen both of the others, we then get a field form with a dimple in the middle; if the centre one be strengthened and the others weakened, we have a peaked field form curve. It follows that by suitable adjustments we can secure that the mean square of the induction remains practically constant over a considerable range while the mean has very different values, and in this way obtain an adjustable ratio between the direct and alternating pressures.

The subjects dealt with in the following sections have been left to the end to avoid breaking the sequence of the treatment.

47. Power Measurements. We will now briefly consider a few ways in which polyphase power can be measured. If it is known

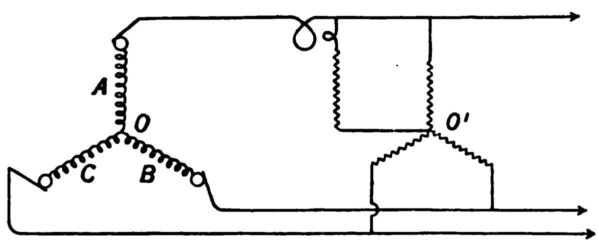


Fig. 78

that the circuits of a polyphase system are balanced, a single wattmeter properly connected to one armature will give the power per phase; but it may be impossible to secure contact with the desired point; thus in a star three-phase load the neutral may not be accessible. This difficulty is overcome by providing an artificial

neutral as shown in Fig. 78, where the shunt, its resistance and one of the star legs must be so adjusted as to give the proper shunt resistance for the instrument: from symmetry, with balanced loads, the instrument will then read one-third of the output. If in addition to assuming balance we assume sine pressures and currents, we can proceed as follows. Suppose that we wish to measure the load in a mesh as shown in Fig. 79. Connect the current circuit of the wattmeter in one of the common mains, and read the successive indications when the shunt coil is across either P or Q. The current in the main is $\sqrt{3}$ times the armature current, I, and if the angle of phase on the load be λ, we know the phase angles between the other main pressures and the current are respectively $(30° - \lambda)$ and $(30° + \lambda)$. It follows that if the main pressure be V, the wattmeter indications are respectively $\sqrt{3}VI \cos(30° - \lambda)$ and $\sqrt{3}VI \cos(30° + \lambda)$. Hence the sum of them gives $3VI \cos \lambda$ or the total power. Similar considerations apply to a star load.

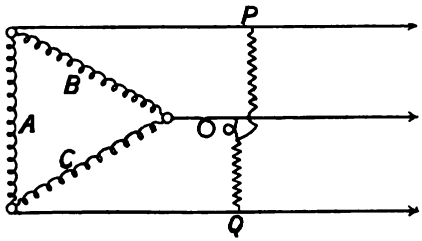


Fig. 79

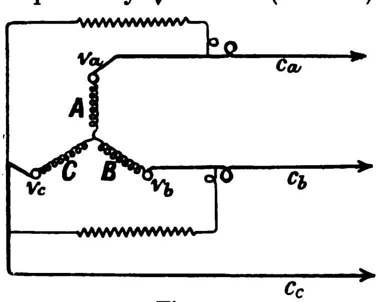


Fig. 80

With two wattmeters it is possible to determine the power even with non-balanced load and non-sine curves. Take the star connection shown in Fig. 80 and at any moment let the pressures relative to the neutral and the currents be as indicated. The power at that moment is necessarily $e_a i_a + e_b i_b + e_c i_c$, so that the mean power is

$$W = \frac{1}{T}\int_0^T (e_a i_a + e_b i_b + e_c i_c)\, dt.$$

But we know that for an open star connection we have $i_a + i_b + i_c$, so that $e_c(i_a + i_b + i_c) = 0$, and hence

$$W = \frac{1}{T}\int_0^T \{i_a(e_a - e_c) + i_b(e_b - e_c)\}\, dt,$$

which is clearly the quantity that the two wattmeters are measuring. For power factors of about 0·5 one of the wattmeter's readings is

negative, and in such comparatively rare cases there is a chance of error. The instruments are often combined into one with both shunt coils fixed to a common spindle, and error cannot then arise. The mesh case follows readily from the above. If the star has its neutral joined, the condition of zero potential of that point will not necessarily be fulfilled, and error will ensue.

48. Losses and Efficiency. A short enquiry into the losses in alternators can now be undertaken.

The sources of loss in alternators do not essentially differ from those in direct current machines, but are somewhat differently proportioned, and as a rule are a little larger. They may be classed under the usual categories of ohmic losses and rotational losses. The ohmic losses are in the excitation of the magnets and in the resistance of the armature; the former is determined from the current necessary to provide the full load excitation, generally derived in the ordinary manner from the saturation and short-circuit curves, and the assigned direct current supply pressure. The armature loss is usually derived indirectly from a measurement of the resistance and the knowledge of the necessary full load current to be delivered, generally at a power factor of 0·8. The rotational losses offer more difficulty; they are usually investigated by means of a calibrated direct current motor which is belted or otherwise geared to the alternator. The mechanical part of the rotational loss is then measured by running the machine unexcited at its proper speed; if it be desired to separate the bearing loss from the windage one this can be done with fair accuracy by measuring the flow of oil to the bearings as in a turbo-alternator, and the rise of temperature produced. The windage loss is very often far higher than in direct current machines, especially so in turbos, where it may be quite large. This loss is, however, incidental to the rejection of the heat produced in the machine, and is therefore a necessary evil. The skill of the designer comes in in providing the maximum cooling effect for the minimum windage loss. The core loss in hysteresis and eddy currents can be found in a similar way with full load excitation, but with no load on the machine, by taking the difference between the power delivered by the motor when the machine is so excited and when non-excited. This loss will also include many parasitic losses as, for example, eddy losses in the polar faces, in the armature windings and the iron near them, in

various places where solid metal is necessary, as for example in the massive rings required to constrain the windings of the rotors in turbos, and indeed in many other more or less obscure places. The total rotational loss, when no separation is required, can be also found by running the machine as a synchronous motor, but the main difficulty is in determining the so-called "stray" losses. These are due to various causes; all the above-mentioned parasitic losses will be differently distributed and mostly augmented by the action of the armature field present on load, and indeed will vary not only with the amount of the load-current but with its phase angle. One very important effect is the distortion of the flux in the poles owing principally to the cross-magnetising effect of the inphase current. This often involves considerable increase in the hysteresis and eddy losses consequent on the increase in maximum induction produced by the shear of the flux. In monophasers there is an additional source of loss; it will be remembered that such machines have a double frequency effect produced by the backwardly rotating armature flux band which causes considerable increase in the losses; further, the pulsation due to this effect acts on the whole magnetic circuit of the machine, as is well shown by the oscillograph records of the field magnet current (Fig. 82), where a double frequency alternating current is seen to be superposed on the steady magnetising current. This pulsation must be acting all through the circuit, though it is often much damped down by a suitable amortisseur grid; hence additional sources of loss are present. These various increments in the parasitic losses are not susceptible of predetermination except from empirical formulae, and are generally allowed for in either of two ways; one is to allot a fictitious increase in the armature resistance sufficient to account for them, which has a basis in the fact that the losses are found to be quite roughly proportional to the square of the armature current. The second method is to take a short-circuit test by means of the coupled motor, and estimate the losses on short-circuit by deducting the mechanical losses from the observed power supplied. From this amount it is usual also to deduct the incident copper loss, and to assume that the remainder is the "stray power." This can only be an approximation, since the distribution of the increments of loss corresponds to a 90° lagging current instead of the actual current on load, which may result in a considerable difference, but in default of a better way of ascertaining the loss, this has to suffice. Sometimes

the copper loss in the test is not deducted. By such observations we can arrive at a conventional method of specifying the efficiency: a common way is to specify that the losses shall be taken as being (1) the excitation loss on full load flux, (2) the armature copper loss found as above, (3) the rotational loss on no load, (4) the stray loss as found above. The efficiency is then to be calculated from the sum of these losses and the assumed output.

Owing to the large size of most modern machines direct efficiency tests are out of the question, and it is as a rule quite impossible to arrange for a Hopkinson test; in the laboratory it is possible to arrange a pair of machines to be so coupled in phase as to circulate full power currents under full excitation, and to measure

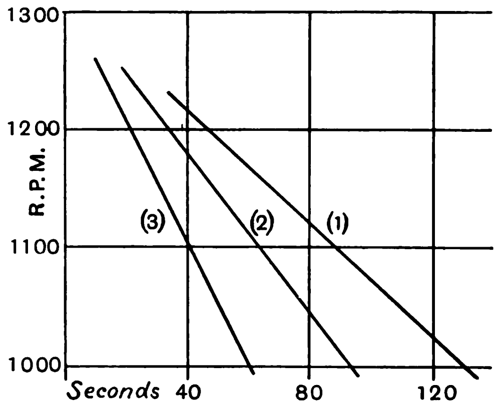


Fig. 81

the losses by a calibrated motor or otherwise, but usually one has to be satisfied with loss measurements. Apart from the indirect tests just considered two other useful methods are available. The first is the ordinary deceleration test, and with turbos this can be used with advantage; from their high speed it is only necessary to determine a small portion of the full curve as is shown in Fig. 81, where (1) is the deceleration curve when unexcited, (2) that when excited and (3) that when delivering a small known load under so small an excitation that only mechanical losses need be considered. The curves are practically straight. In any one of them, if L be the couple producing the deceleration and K the moment of inertia of the machine, we have $L = K\, d\omega/dt$. In the first (1) this couple is merely the mechanical couple, in the third (3) it is that couple

increased by a known one, namely the couple expressed by the extra constant load in foot pounds per second divided by the angular velocity at the point of test. It follows that from (1) and (3) an approximate evaluation of K can be made and hence all the couples and corresponding losses can be found. The moment of inertia can also be calculated with some expense of time from the drawings. This method is a very good one for determining the losses on no load, but cannot give much information as to the stray loss. The following method has the advantage that it can be applied to a machine which is actually working in the station under load conditions. Most modern turbo-alternators are cased in and provided with forced air cooling, hence the heat rejected by the cooling air can be found if we can measure the flow of air per second and the rise of temperature it acquires in passage through the machine. The quantity of air can be measured volumetrically by finding the average velocity with which it leaves the final orifice, which is of known dimensions. This velocity is measured either by suitably calibrated anemometers or by specially designed apparatus depending on measuring the cooling effect of the current of air when passing a series of fine wires carrying a steady current; the temperature difference produced by the draught of air is measured by the difference of resistance between wires in the draught and others not in it. The temperature of the air itself is measured either with ordinary thermometers or by electric ones. By making suitable barometric and humidity corrections a very good determination of the rejected heat can be made. To this must be added an estimate of the comparatively small amount of heat rejected by general radiation and convection from the whole casing, which can readily be allowed for from other tests. With careful work it is probable that this method is capable of giving a very accurate measurement of the losses under load as well as on no load.

49. Sudden Short-circuit. If an alternator be short-circuited, run up to speed, and then gradually excited, it will give a definite current at full excitation which is equal to the full E.M.F. for that excitation divided by the synchronous impedance; this is generally somewhere about three or four times the normal full load current. But if it be fully excited and then suddenly short-circuited, a current many times as large will flow for the first few periods as is shown in Figs. 82*a*, and *b*, which give the short-circuit

current and below it the field magnet current, of a machine thus short-circuited. As a preliminary to an explanation of these curves, let us first take the simple case of a circuit whose self-inductance is L and resistance R being suddenly placed across mains whose pressure is given by $e = E \sin \omega t$. The equation giving the current is

$$L \frac{di}{dt} + Ri = E \sin \omega t.$$

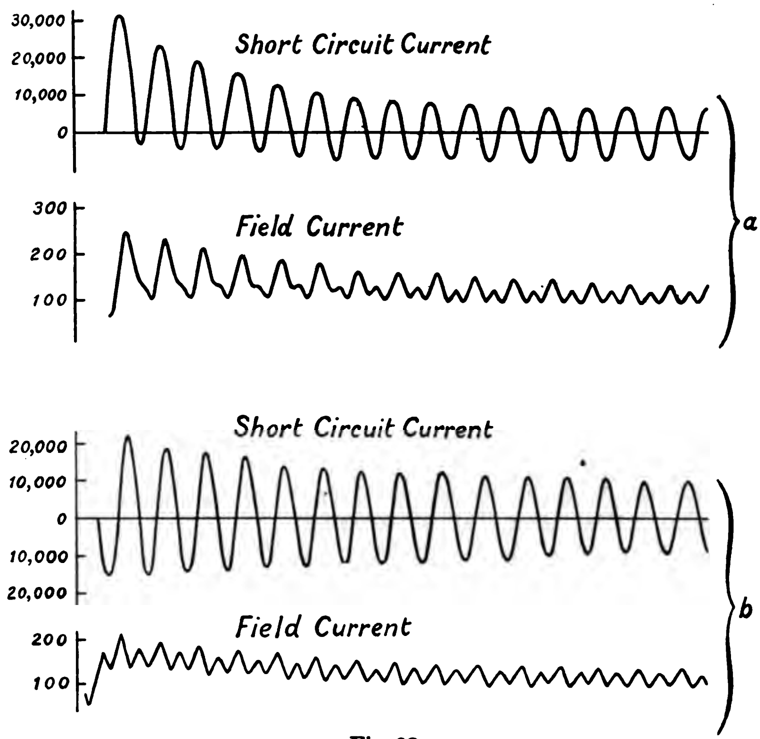


Fig. 82

Now such an equation has two parts in its solution, the "particular integral" and the "complementary function." Hitherto we have only been concerned with the former and we know that this leads to the solution

$$i = \frac{E}{(L^2\omega^2 + R^2)^{\frac{1}{2}}} \sin (\omega t - \lambda) \quad \text{or} \quad i = I \sin (\omega t - \lambda),$$

where λ is the ordinary lag angle. But this solution has no constants of integration which are all carried by the complementary function,

and are necessary in this problem inasmuch as we have to fit in certain initial and final conditions, namely, that at the instant of making the circuit there is no current, and that eventually the current settles down to the equation given above. Now the complementary function is a solution of the equation with the right-hand part zero, namely a solution of $L\frac{di}{dt} + Ri = 0$, but this gives $i = AE^{-\frac{L}{R}t}$. Let us reckon the time from the moment at which the E.M.F. is zero, and let the instant of make be t_0 seconds thereafter. At that instant we must have zero current, so let us write

$$i = I\sin(\omega t - \lambda) - I\epsilon^{-\frac{L}{R}(t-t_0)}\sin(\omega t_0 - \lambda),$$

where $I = E/Z$. It will be seen that this satisfies the required conditions since at time $t = t_0$ there is no current, and after a very long time it reduces to $I\sin(\omega t - \lambda)$. Since $\sin(\omega t_0 - \lambda)$ is constant this means that superposed on the normal alternating current we have an exponentially falling steady current which will have zero value if we just hit off the time of make so that t_0 is λ/ω, namely, the instant when the E.M.F. is zero, and may have any initial value up to a maximum E/Z which will be the value if we make circuit at the moment of maximum E.M.F. Compare this with the above figures, and one sees there is a rough general agreement; there is a sine current in the armature superposed on an exponential steady one, which may be absent as in Fig. 82 *b*, but there is one great difference inasmuch as the sine current has not its final steady value throughout but starts off much larger and has itself a decreasing maximum slowly falling to the steady value.

Instead of suddenly making a circuit on the above coil, suppose that we suddenly short-circuit a generator. If we first postulate that the flux from the field shall be entirely unaffected by the armature, the effect would be the same as with the first coil provided we substitute the true resistance and reactance of the armature for those of the coil. For simplicity let us suppose that all the phases are simultaneously short-circuited. Then as far as the sinoidal part of the armature currents is concerned, they will as usual produce a sudden belt of current round the armature which is stationary in space, and since the armature has high reactance, this band of current will produce a belt of demagnetising turns opposing the coils on the field. If the exponential steady current is present, it will produce also a magnetising force, but

this will be fixed relative to the armature since it is a steady current, and will hence rotate with it being just as if the armature were temporarily a field magnet separately excited. This virtual field magnet then rotates relative to the poles and will consequently generate currents in any circuits round or in them, for example it will generate in the field magnet windings an alternating current of the *same* frequency as that of the machine. Thus if an exponential term occurs at short-circuit we expect to find associated with it a pronounced ripple in the field magnet current of the *fundamental* frequency, if it is not present, that should be practically absent. An inspection of the figures shows that this is true; the top one shows a ripple of double frequency like the bottom, due to a cause to be mentioned later, but the top one gives clear evidence of the existence of a *fundamental* ripple as well. But in addition to the effect of the exponential steady current we have also to consider that of the suddenly created stationary band demagnetising the field. It is impossible for the field flux to change suddenly in response to this demand, owing to the magnetic inertia possessed by it. An opposing band of eddy currents, or currents induced partly as eddies, partly in the field winding, must arise with great swiftness, the speed of rise being settled by the self-inductance and resistance of the eddy current paths say L_1 and R_1. In a fraction of a second we then have established this new belt of current in and round the poles. If the eddy circuits were entirely devoid of resistance this current would flow on unchanged; it is as if it had been established by an electromagnetic "blow." Under such circumstances the flux produced by the field is largely augmented and hence the amplitude of the armature E.M.F. is increased. But this eddy will slowly die away and with it will fall off the extra excitation according to some such law as an exponential $I^{-\frac{L_1}{R_1}t}$, although from the nature of the case neither R_1 nor L_1 can be expected to be constant. This then accounts for the fact that the sinoidal part of the armature current, being due to the new flux, falls off also in an exponential manner and is not constant as in the simple case of the coil. The presence of the exponential current in the fields is shown in both the figures by the gradual fall in the maximum current.

So far we have supposed that the machine was shorted on all its phases at once, which is very unlikely. If only one be shorted, the machine is really acting as a monophase alternator. Now we

know that in such a machine the armature alternating effect can be looked on as being made up of two equal rotating ones, one of which is brought to rest relative to the poles, and the other rotates at twice the speed. With respect to the first component the above still holds, while the latter is responsible for the appearance all through the field current curves of the ripple of double frequency that is seen in them.

The eddy current produced is of enormous magnitude, since it tends to be equal to the ampere wires under the pole at the moment, and these may be over 100,000. It produces very powerful effects, even rings of flame have been seen round the ends of the field magnet on short-circuit. But the most important effect is the excessively large mechanical forces brought to bear on the armature wires with currents some 20 times the normal full load current; as the force varies as the square of the current, it may readily amount to as much as 1000 pounds per foot run; further the force is so sudden as to amount to an impact, which doubles its effect. Machines with many ampere-turns per phase such as turbo-alternators are especially difficult to protect, and the mechanical support of the end wires of such machines' rotors requires the most careful design.